

MATH 221 — THE FIRST MIDTERM

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Your TA: (circle one)

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Problem	Score
1 (a)	
1 (b)	
1 (c)	
2 (a)	
2 (b)	
3 (a)	
3 (b)	
4	
5	
Total	

(1) (a) If one defines a function by saying that

$$y = f(x) \iff y \text{ is the largest solution of } y^2 + 6y = x$$

then what is the domain of f , and find a formula for $f(x)$.

(b) Does there exist a function f such that $f(2x+4) = x \sin x$ holds for all real numbers x ? If there is such a function, then find the domain of f and a formula for $f(x)$.

(c) Does there exist a function f such that $f(x^2+4) = x$ holds for all real numbers x ? If there is such a function, then find the domain of f and a formula for $f(x)$.

(2) (a) State the ϵ - δ definition of " $\lim_{x \rightarrow a} f(x) = L$ ".

(b) Show, using the ϵ - δ definition, that $\lim_{x \rightarrow 2} \frac{12}{x+2} = 3$.

(3) (a) Compute the following limits using the limit properties:

$$\lim_{x \rightarrow 2} \frac{x - \frac{4}{x}}{x^2 - 2x} =$$

$$\lim_{x \rightarrow 0} \frac{\sin \pi x}{\tan 3x} =$$

$$\lim_{t \rightarrow 0} \frac{2 - \sqrt{4 + t^2}}{t^2} =$$

(c) Show that the limit properties imply that the limit

$$\lim_{x \rightarrow 0} \frac{x+1}{x(x+3)}$$

does not exist.

(4) For which values of the constants a and b is the function

$$f(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ a + x^2 & \text{for } 0 < x \leq 2 \\ b/x & \text{for } x > 2 \end{cases}$$

continuous?

(5) Use the definition of the derivative as a limit to find

(a) $f'(1)$ if $f(x) = \sqrt{x+3}$.

(b) $g'(2)$ if $g(x) = x^2 + \frac{2}{x}$.