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CALCULUS 221

FINAL EXAM

I. M. Isaacs

Wednesday December 20, 2006

7:45 – 9:45 A.M.

Do all problems — 200 points.
Use backs of pages for scrap, or if you need more space.

NAME: ______________________________

TA: ______________________________

Do not write below here.

Prob. 1: __________ out of 42.

Prob. 2: __________ out of 56.

Prob. 3: __________ out of 39.

Prob. 4: __________ out of 32.

Prob. 5: __________ out of 15.

Prob. 6: __________ out of 16.

Total: __________ out of 200.

1. (42 POINTS.) Compute dy/dx at the specified point for each of the following.

(a) $y = \dfrac{xe^{-x^2}}{\sqrt[3]{9 - x^2}}$ at the point where $x = 1$.

(b) $x = (t + 1)\sin(t)$ and $y = e^{2t}$ at the point where $x = 0$, $y = 1$.

(c) $y = f(x)$ at the point where $x = 1$, given that $f(x)^2 + 2 = f(x^2) + 4x$ and that $f(1) = 2$.

2. (56 POINTS.) Integrate.

(a) $\displaystyle\int_0^1 \frac{\ln(2x+1)}{2x+1}\,dx$

(b) $\displaystyle\int_0^1 x^3(1-x^2)^{10}\,dx$

Problem 2 continued: Integrate.

(c) $\int \sec^2(3x)\tan(3x)\,dx$

(d) $\int_0^{\ln(3)/2} \frac{e^x\,dx}{1+e^{2x}}$

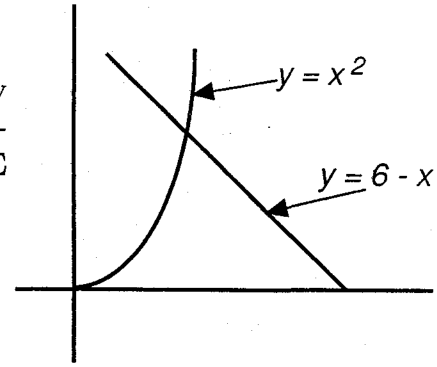
3. (39 POINTS.) Calculate each of the following.

(a) $\lim_{x \to 1} \frac{\ln(x) - x + 1}{(x-1)^2}$

(b) The approximate value of $\sqrt[4]{15}$ based on a linear approximation (also called a tangent line approcimation) of the function $f(x) = x^{1/4}$, starting from the point where $x = 16$.

(c) $\log_b(a)$, given that $3\ln(a) = 2\ln(b)$.

4. (32 POINTS.) Consider the roughly triangular region bounded by the curve $y = x^2$, the line $y = 6 - x$ and the positive x axis. Use integrals to express the following volumes, but **DO NOT EVALUATE** your integrals.



(a) The volume obtained by rotating the region about the x-axis.

(b) The volume obtained by rotating the region about the y-axis.

5. (15 POINTS.) Let $f(x) = e^x - 2x$ for all x.

(a) Does $f(x)$ take on a **maximum** value anywhere? Either find all x where $f(x)$ is maximum and find the maximum value, or else explain briefly why no maximum exists.

(b) Does $f(x)$ take on a **minimum** value anywhere? Either find all x where $f(x)$ is miniimum and find the minimum value, or else explain briefly why no minimum exists.

(c) Decide whether or not the graph of $y = x^2$ ever crosses the graph of $y = 2x$ and explain briefly how you know.

6. (16 POINTS.) The graph of $y = f(x)$ goes through the point $(1, 2)$ and has the property that the normal line to the curve at each at each point (x, y) on the curve crosses the x-axis at the point $(x + 1, 0)$. Compute $f(7)$. HINT: Observe that the graph can never cross the x-axis.

Have a pleasant holiday season.

THE END