Math 221 Calculus
Midterm Three
Instructor: Alexander Kiselev

No calculator allowed.
Write detailed work to obtain full credits.

Your Name:

Please circle your TA's name:

Ulysses Andrews	Matt Davis	Jie Ling	Kim Schattner

Problem/pt	Score
P1/15	
P2/20	
P3/15	
P4/10	
P5/15	
P6/15	
P7/10	
Total/100	

Math 221, Fall 2006, Lecture 6, Midterm 3.

Try to do first the problems that look simplest to you. You don't have to follow the order. Avoid spending too much time on one question with others not done yet. Good luck!

1. Compute

a. (10) The indefinite integral $\int \frac{x \arcsin x^2}{\sqrt{1-x^4}}\, dx$

b. (5) The derivative of $F(x) = \int_0^{\log(2+\cos x)} e^t\, dt$.

2. (20) Sketch the graph of a function $f(x) = 2x^3 - 9x^2 - 60x + 11$. To generate the sketch, find critical points, classify local extrema, inflection points, intervals of increase, decrease and up/down concavity. How many roots (zeroes) does f have? Justify your answer.

3. (15) The producer of ice cream wants to figure out the shape of a conical cap that has maximum volume for a given value of surface area (waffle used). A conical cap is defined by its height, h, and radius of the foundation, r. The surface area of the conical cap is equal to $\pi r\sqrt{r^2+h^2}$, and should be fixed - let us set it equal to 1 for simplicity. The volume is equal to $\frac{1}{3}\pi r^2 h$. What should r equal to in order for the volume to be maximal?

4. (10) State L'Hopitale's rule. Compute $\lim_{x\to 0} \frac{x\log(\cos x)}{x-\sin x}$.

5. a. (10) State and prove the Mean Value Theorem. You may use without proof other properties and theorems presented in lectures, but you should always indicate very clearly what is it that you are using.

b. (5) State at least three different properties of definite integrals (you do not have to prove them, just state).

6. (15) Describe what is called a partition of $[a, b]$ and what is a Riemannian sum corresponding to this partition. Consider the function $f(x) = \frac{1}{1+x^3}$. Compute the upper and lower Riemannian sum for this function on interval $[0, 1]$ using the partition $P = \{0, 1/3, 2/3, 1\}$.

7. (10) Compute the area bounded by the curves $y = \sqrt{x}$, $0 \le x \le 1$, $y = \sin(\frac{\pi}{2}x)$, $1 \le x \le 2$ and $y = 0$.