Math 221 Calculus, Final Exam
Instructor: Alexander Kiselev

No calculator allowed.
Write detailed work to obtain full credits.

Your Name:

Please circle your TA's name:

Ulysses Andrews	Matt Davis	Jie Ling	Kim Schattner

Problem/pt	Score
P1/15	
P2/20	
P3a/20	
P3b/10	
P4/30	
P5/25	
P6/25	
P7a/15	
P7b/15	
P8/25	
Total/200	

Math 221, Fall 2006, Final Exam.

Try to do first the problems that look simplest to you. You don't have to follow the order. Avoid spending too much time on one question with others not done yet. Good luck!

1. (15) Compute the volume obtained by rotating the graph $y = \frac{x}{\sqrt{x^3+1}}$, $0 \leq x \leq 2$ around y axis.

2. (20) Consider the curves $y = 0$, $x = 0$, $x = 2$, $y = x^2 + 1$ and $y = x + 1$. These curves split the plane into several regions; find the area of the region containing the point $(1, 1)$.

3(a). (20) State the Fundamental Theorem of Calculus. Prove the first part of the Theorem.

3(b). (10) Compute the derivative of $F(x) = \ln(\int_0^{\arcsin x} e^t \, dt)$.

4. (30) Consider the function $f(x)$ given by

$$f(x) = \begin{cases} -x^2, & x < -2 \\ -4, & -2 \leq x < -1 \\ x, & -1 < x \leq 0 \\ \frac{1-\cos x}{\sin x}, & x > 0 \end{cases}$$

At which points a the limit $\lim_{x \to a} f(x)$ does not exist? At which points $f(x)$ is not continuous? At which points $f(x)$ is not differentiable? Justify your answers.

5. (25) Let $f(x) = \frac{1}{1+(\cos x)^2}$. Compute the upper and lower Riemannian sums for this function on the interval $[0, \pi]$ using the partition $P = \{0, \pi/4, \pi/2, 3\pi/4, \pi\}$. Is $f(x)$ Riemann integrable or not? How do you know?

6. (25) A ladder 4 meters long rests against a vertical wall. Assume that the bottom of the ladder is sliding away from the wall with the velocity one meter per second. What is the rate of change of the angle θ between the top of the ladder and the wall when $\theta = \pi/4$ radians?

7(a) (15) A curve is defined by $x(t) = e^t + e^{-t}$, $y(t) = 2t$, $-2 \leq t \leq 2$. Find the length of this curve.

7(b) (15) For the same curve defined by $x(t) = e^t + e^{-t}$, $y(t) = 2t$, $-2 \leq t \leq 2$, find the equation of the tangent line at a point corresponding to $t = 1$.

8. (25) A barrel has a shape of a cylinder (with both top and bottom present). What is the maximal volume of the barrel that can be made using one square meter of material?