MATH 221 (3)

(8 pgs)

MATH 221 - SATELLITE LECTURES
FIRST MIDTERM

YOUR Name: _____

YOUR Instructor's Name: _____

PROBLEM	VALUE	SCORE
I	15	
II	15	
III	15	
IV	10	
V	16	
VI	15	
VII	14	
TOTAL	100	

- JUSTIFY YOUR ANSWERS: Answers without supporting work will not receive full credit.
- SIMPLIFICATION: It is not necessary to simplify your answers.
- GOOD LUCK and have a great exam!

JUSTIFY YOUR ANSWER!!!
–ANSWERS WITH LITTLE OR NO SUPPORTING WORK–
WILL RECEIVE LITTLE OR NO CREDIT

I. (15 pts)

(a) If $f(x) = \dfrac{x\sin(x)}{x+1}$, find $f'(x)$.

(b) If $g(x) = \cos(\sin(x^2))$, find $g'(x)$.

(c) If $h(t) = \sec(\sqrt{t-1})$, find $\dfrac{dh}{dt}$ when $t = 2$.

JUSTIFY YOUR ANSWER!!!
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II. (15 pts)

(a) $\lim_{x \to -\infty} \dfrac{3x^3 + 1}{-x^3 + 6x} =$

(b) $\lim_{x \to 0} \dfrac{\sin(2x)}{\sin(3x)} =$

(c) $\lim_{x \to \pi} \dfrac{\cos(x) + 1}{x - \pi} =$

(HINT: Think of the form of this limit.)

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III. (15 pts)

(a) Find a vertical asymptote to the graph of

$$f(x) = \frac{\sqrt{3x^2+5}}{x-7}..$$

(b) Find a horizontal asymptote to the graph of

$$f(x) = \frac{\sqrt{3x^2+5}}{x-7}.$$

(c) Show that there is at least one solution to the equation $x^4 - 3x + 1 = 0$.

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IV. (10 pts) Using the limit definition and properties of limits, find $f'(2)$ where $f(x) = \sqrt{3x}$.

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V. (16 pts) Let $f(x) = \sqrt{|x|}$ i.e. $f(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ \sqrt{-x} & x < 0. \end{cases}$

(a) Using limits, show $f(x)$ is continuous at $x = 0$.

(b) Using limits, show $f'(0)$ **does not** exist.

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VI. (15 pts) Regard y as a function of x in the relation $x^2 - y^2 = 4$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) Show that the point $(-3, -\sqrt{5})$ lies on the curve given by the relation.

(c) Find the equation of the **normal line** to that curve at the point $(-3, -\sqrt{5})$.

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VII. (14 pts) In optics three quantities $f > 0$, $p > 0$ and $q > 0$ are related by the equation $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$. Suppose that f is constant.

(a) Find the rate of change of p with respect to q.

(b) For what values (if any) of q is p an increasing function of q.