Craciun

Math 221 – MIDTERM EXAM 2 – Fall 2005

YOUR NAME: ______________________________

Circle your TA's name and the meeting time of your Discussion Session:

Julien Colvin	11:00am	3:30pm
Matthew Davis	7:45am	9:55am
Guillermo Mantilla	12:05pm	2:25pm
Ravaris Moore	9:55am	
Kathryn Rouse	8:50am	9:55pm

Show all your work. Answers without showing all work receive zero points.

1. *(30 points)* Compute the following antiderivatives:

(a) $\int (x^2 + \pi)dx$

(b) $\int \frac{1}{\sqrt{x^3}}dx$

(c) $\int 3x^2(x^3 - 3x)dx$

(d) $\int (\sin\theta - \cos\theta)d\theta$

(e) $\int (\sqrt{2}x + 1)^3\sqrt{2}dx$

(f) $\int (5x^2 + 1)(5x^3 + 3x - 8)^6 dx$

2. *(10 points)* Use differentials to compute an approximate value of $\sqrt[3]{26.91}$.

3. *(10 points)* Consider the function $g(x) = 3x^4 - 4x^3 + 2$. Determine where the graph of this function is increasing, decreasing, concave up, and concave down; then, sketch the graph.

4. *(15 points) (a)* Identify the critical points and find the maximum value and minimum value of the function $f(x) = x^3 - 3x + 1$ on the interval $[\frac{3}{2}, 3]$.

(b) What number exceeds its square by the maximum amount? *Hint: Begin by explaining why this number has to be inside the interval* $[0, 1]$. *Then, solve the problem by finding the maximum value of a function on the interval* $[0, 1]$.

5. *(30 points)* Compute the following sums:

Hint: for some of them you might use $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$.

(a) $1+2+3+4+\cdots+41 = ?$

(b) $2^2+4^2+6^2+8^2+\cdots+100^2 = ?$

(c) Find the value of the collapsing sum $\sum_{i=1}^{30}\left(\frac{1}{i}-\frac{1}{i+1}\right)$.

(d) $\sum_{k=1}^{20}(k^3 - k^2) =?$

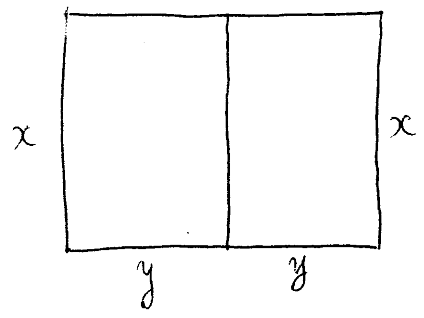
(e) $\sum_{k=21}^{50}(k^3 - k^2) =?$

(f) If $\sum_{i=1}^{10} a_i = 40$ and $\sum_{i=1}^{10} b_i = 50$, what is the value of $\sum_{i=1}^{10}(3a_i + 2b_i)$?

6. *(15 points) (a)* Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x}{y}$; then find the particular solution that satisfies the condition $y = 1$ at $x = 1$.

(b) If the brakes of a car, when fully applied, produce a constant deceleration of 11 feet per second per second, what is the shortest distance in which the car can be braked to a halt from a speed of 60 miles per hour? *Hint: set up a differential equation by using the fact that "deceleration" means negative acceleration; also, use the fact that 60 miles per hour is the same as 88 feet per second.*

7. *(10 points)* A farmer wishes to fence off two identical adjoining rectangular pens, each with 900 square feet of area, as shown in the figure below. What are x and y so that the least amount of fence is required?



8. *(15 points)* *(a)* Consider the function $f(t) = t^2$ on the interval $[0, 2]$. Apply the Mean Value Theorem to find some c such that $\frac{f(2)-f(0)}{2-0} = f'(c)$. Sketch the graph of $f(t)$ on the interval $[0, 2]$, and explain the geometric interpretation of the Mean Value Theorem on your graph.

(b) Suppose that in a race horse A and horse B begin at the same point and finish in a dead heat. Prove that their speeds were identical at some instant of the race.

9. *(15 points) (a)* The total area of the n^{th} "inscribed polygon" that approximates the region under the graph of $y = x^2$ on the interval $[0, 5]$ is given by the formula $\boxed{A_n = \sum_{i=1}^{n} \left[\frac{5}{n} \cdot (i-1)\right]^2 \cdot \frac{5}{n}}$. Use this formula to find the exact value of the area A of the region under the curve $y = x^2$ on the interval $[0, 5]$.

(b) Calculate the Riemann sum $\boxed{\sum_{i=1}^{n} f(\bar{x}_i)\Delta x_i}$ for the function $f(x) = x^3 + 1$, the partition $P: -2 < 0 < 1 < 2 < 4$, and the sample points $\bar{x}_1 = -1, \bar{x}_2 = 0.2, \bar{x}_3 = 1.6, \bar{x}_4 = 3$.