Exam 1

CALCULUS 221

6th WEEK EXAM

I. M. Isaacs
Thursday, February 27, 2003
5:30 – 7:00 P.M.

Do all problems — 100 points.
Use backs of pages for scrap, or if you need more space.

NAME: _____

TA: _____

Do not write below here.

Prob. 1: _____ out of 24.

Prob. 2: _____ out of 24.

Prob. 3: _____ out of 6.

Prob. 4: _____ out of 6.

Prob. 5: _____ out of 20.

Prob. 6: _____ out of 12.

Prob. 7: _____ out of 8.

Total: _____ out of 100.

1. [24 POINTS] Compute the derivative y' for each of the following.

(a) $y = \dfrac{\pi^3}{x^5}$.

(b) $y = \dfrac{x^2}{(2-3x)^5}$.

(c) $y = \csc^2(x^3)$.

(d) $y = \sqrt[5]{1-2x^2}$

2. [24 POINTS] Compute the following.

(a) du/dv, where $u = y^2$ and $v = y + 3y^3$.

(b) dy/dx, where $y = f(1 - 2x)$ and $f'(x) = \sec(x)$.

(c) dy/dx at the point $(1, 2)$, where $y = y^2 - 2x^3$.

(d) d^2y/dx^2 at the point $(1, 2)$, where $y = y^2 - 2x^3$.

3. [6 POINTS]

(a) Find a function $f(x)$ such that $f'(1) = \lim_{h \to 0} \dfrac{\frac{(1+h)^4}{2+h} - \frac{1}{2}}{h}$.

(b) Evaluate the limit in Part (a).

4. [6 POINTS]

The function $f(x)$ is defined as follows for $x > 0$: $f(x) = \begin{cases} 1/x & \text{if } x \neq 2. \\ 0 & \text{if } x = 2. \end{cases}$

(a) Does $\lim_{x \to 2} f(x)$ exist? If not, say so and explain why, and otherwise, compute the limit.

(b) Compute the quantity $|f(x) - (1/2)|$ when $x = 2$.

(c) Let $\epsilon = 1/4$. Does there exist a number $\delta > 0$ such that $|f(x) - (1/2)| < \epsilon$ for all values of x such that $0 < |x - 2| < \delta$? If not, say so, and otherwise compute the largest possible value of δ.

5. [20 POINTS] The following table gives some values for the function $f(x)$ and its derivative $f'(x)$. Use these data to compute the following items. If there is not enough information given to determine some item, write "UNKNOWN".

x	1	2	3	4
$f(x)$	-1	3	1	5
$f'(x)$	3	2	-2	7

(a) $f(f(x)+1))$ at $x=2$.

(b) $g'(2)$, where $g(x)=f(f(x))$.

(c) $h'(2)$, where $h(x)=f(x)^2$.

(d) dy/dx at $x=2$, where $y=f(x^2-3)$.

(e) $k'(2)$, where $k(x)=\dfrac{f(x^2)}{f(x)}$.

6. [12 POINTS] A particle moves along the x-axis in such a way that at time t, its position is given by $x = 1 + 32t - t^4$, where t is measured in seconds and the units on the x-axis are in centimeters.

(a) Find the velocity of the particle when $t = 0$ and when $t = 1$.

(b) Find the average velocity of the particle during the time interval $0 \leq t \leq 1$.

(c) The particle moves to the right for a while, reaches some farthest right point, and then starts turning to the left. What is the particle's velocity at the moment when it is at its farthest right point? When and where does that occur?

(d) What is the acceleration of the particle when it is at its farthest right point.

7. [8 POINTS] The point $(1, 2)$ lies on the curve described by the equation $x^2 - xy + y^2 = 3$. Find the point other than $(1, 2)$ where the normal line to the curve at $(1, 2)$ crosses the curve.

(RECALL: The **normal line** to a curve at a point is the line through that point and perpendicular the tangent at that point.)

THE END