EXAM 2

CALCULUS 221

11th WEEK EXAM

I. M. Isaacs
Thursday, April 10, 2003
5:30 – 7:00 P.M.

Do all problems — 100 points.
Use backs of pages for scrap, or if you need more space.

NAME: _____

TA: _____

Do not write below here.

Prob. 1: _____ out of 35.

Prob. 2: _____ out of 10.

Prob. 3: _____ out of 10.

Prob. 4: _____ out of 7.

Prob. 5: _____ out of 16.

Prob. 6: _____ out of 11.

Prob. 7: _____ out of 11.

Total: _____ out of 100.

1. [35 POINTS] Integrate these.

(a) $\int_{-1}^{2} (2 - 3x^2)\, dx$

(b) $\int \frac{\sin(3\theta)}{\cos^2(3\theta)}\, d\theta$

(c) $\int \frac{1 - 2s}{\sqrt{s}}, ds$

There are more integrals on the next page.

Problem 1, continued.

(d) $\displaystyle\int_{-1}^{3} \frac{(t+1)\,dt}{\sqrt{t^2+2t+10}}$

(e) $\displaystyle\int \frac{(1+\frac{1}{x})^{10}}{x^2}\,dx$

5. [16 POINTS]

(a) Let $y = f(x)$ and suppose that $\dfrac{dy}{dx} = y^2 \sin(x)$. If $f(\pi/2) = 1/3$, find $f(\pi)$.

(b) Suppose $d^2s/dt^2 = \sin(t)$ and that when $t = 0$, we have $\dfrac{ds}{dt} = 3$ and $s = 2$. Find the value of s when $t = \pi/2$.

6. [11 POINTS] A hole is bored straight downward from the surface of the Earth to the center, and then a stone is dropped (with initial velocity zero) into the hole. The problem is to calculate the speed of the stone when it reaches the center of the Earth. (We ignore air resistance, intense heat and erupting volcanoes.)

The fact we need from physics is that as the stone falls, its gravity-induced acceleration is proportional to the distance to the center of the Earth. In other words, $a = -ky$, where a is acceleration and y is the distance to the center.

Here is an outline of how to do this problem. First, express k in terms of R and g, where R is the radius of the Earth and g is the the gravitational acceleration at the surface. Next, write the appropriate differential equation involving y, the velocity v and time t. Then, eliminate t from the equation by using the fact that $\dfrac{dv}{dt} = \dfrac{dv}{dy}\dfrac{dy}{dt} = \dfrac{dv}{dy}v$. Finally, solve the differential equation to find the velocity of the falling stone when it reaches the center of the Earth. (The answer should be left in terms of R and g.)

7. [11 POINTS] In an unfortunate accident, $100\,\text{ft}^3$ of oil is spilled at sea. The oil floats on the water forming an oil slick of uniform thickness. (In other words, at each moment, the thickness of the slick is the same everywhere.) When the area of the slick is $10,000\,\text{ft}^2$, the area is expanding at a rate of $30\,\text{ft}^2$ per hour. How fast is the thickness of the oil slick changing at that moment.
HINT: The volume of the oil slick is its area times its thickness. The shape does not matter.

THE END