

# CALCULUS 221

## FINAL EXAM

I. M. Isaacs  
Wednesday May 14, 2003  
5:05 – 7:05 P.M.

Do all problems — 200 points.  
Use backs of pages for scrap, or if you need more space.

NAME: \_\_\_\_\_

TA: \_\_\_\_\_

Do not write below here.

Prob. 1: \_\_\_\_\_ out of 44.

Prob. 2: \_\_\_\_\_ out of 44.

Prob. 3: \_\_\_\_\_ out of 27.

Prob. 4: \_\_\_\_\_ out of 20.

Prob. 5: \_\_\_\_\_ out of 14.

Prob. 6: \_\_\_\_\_ out of 17.

Prob. 7: \_\_\_\_\_ out of 17.

Prob. 8: \_\_\_\_\_ out of 17.

Total: \_\_\_\_\_ out of 200.

1. (44 POINTS.) Compute  $dy/dx$  for each of the following. (If a particular point is indicated, compute the value of  $dy/dx$  at that point.)

(a)  $y = \frac{1}{\sqrt{1-3x^2}}$ .

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(b)  $xe^y = x^2 + 2y$  at the point  $x = 1, y = 0$ .

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(c)  $y = \int_0^{\ln(x)} \sec(t^2) dt$ .

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(d)  $y = \frac{e^{x^2} \sqrt{1-x}}{(1-2x)\sqrt[3]{1+x^2}}$

2. (44 POINTS.) Integrate.

(a)  $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

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(b)  $\int \frac{dx}{1+4x^2}$

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$\int_2^3 \frac{x dx}{3-x^2}$

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(d)  $\int \frac{1+2x}{\sqrt{x}} dx$

3. (27 POINTS.)

Calculate each of the following.

(a)  $\lim_{x \rightarrow \pi/2} \frac{\sin(x) - 1}{(2x - \pi)^2}$

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(b) Find  $f'(2)$ , given that  $f(x) = h(e^x)$  and  $h'(x) = \ln(x)$ .

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(c) Find  $f(\pi)$ , given that  $f'(x) = \sin(x)$  and  $f(0) = 3$ .

4. (20 POINTS.)

- Consider the region bounded by the  $x$ -axis and the curve  $y = \sin(x)$  for  $0 \leq x \leq \pi$ . Set up the integrals but **DO NOT EVALUATE** them for these volume problems.
  - ) The volume when the region is rotated about the  $y$ -axis.

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- ) The volume when the region is rotated about the line  $y = 1$ .

5. (14 POINTS.)

(a) Write an integral equal to the length of the part of curve  $y = \ln(x)$  between  $x = 1$  and  $x = 3$ . **DO NOT EVALUATE** the integral.

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(b) Find numbers  $a$  and  $b$  such that the length of the part of the curve  $y = e^x$  between  $x = a$  and  $x = b$  is exactly equal to the length in part (a).

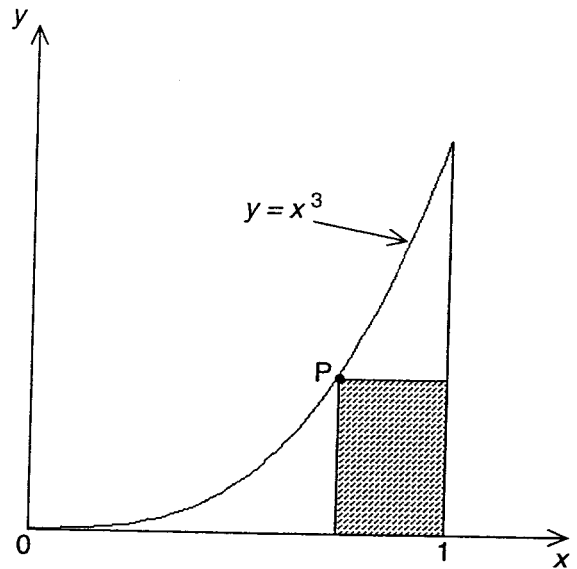
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6. (17 POINTS.)

Suppose that  $(x - 2)\frac{dy}{dx} = y^2$  and that  $y = 1$  when  $x = 0$ . Find the value of  $y$  when  $x = 1$ .

7. (17 POINTS.)

A point  $P$  is chosen on the part of the curve  $y = x^3$  with  $0 \leq x \leq 1$ . Horizontal and vertical lines are drawn through  $P$ , and these lines, together with the  $x$ -axis and the line  $x = 1$  form a rectangle, as shown. Find the coordinates of the point  $P$  so that the area of this rectangle will be as large as possible.



8. (17 POINTS.)

A weightless (but very strong) rope extends from the moon to the surface of the Earth, and this rope is used to lift an object from the Earth's surface into space. The object weighs 1000 pounds on the ground and it is lifted to an altitude 20000 miles above the surface of the Earth. How much work is done? (For this problem, work should be expressed in units of **mile-pounds**.)

Recall that as the object gets farther from the Earth, the gravity it feels gets weaker, and so the weight of the object decreases. In fact, the weight is inversely proportional to the square of the distance from the center of the Earth. When the object starts (at the Earth's surface) its distance from the center of the Earth is, of course, the radius of the Earth, which is 4000 miles.

Have a pleasant summer!