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JOHNSON
VERSION II

221 FINAL - MAY 11, 2005

NAME: _____

T. A.: _____

INSTRUCTIONS: Show all your work. Answers alone will receive little or no credit. Read the instructions carefully and do what is asked. If you do not understand what is being asked, raise your hand and someone will assist you. Calculators and one 8.5" by 11" sheet of paper are permitted.

1. _____

1. _____

2. _____

2. _____

3. _____

3. _____

4. _____

4. _____

5. _____

5. _____

6. _____

6. _____

7. _____

8. _____

TOTAL _____

PART I

1. (15 p'ts.) A particle P is moving along the graph of $y = \sqrt{x^2 - 4}$; $x \geq 2$. The x coordinate of P is increasing at the rate of 5 units per second. How fast is the y coordinate of P increasing when $x = 3$?

Ans. _____

2. (15 p'ts.) The positions of two particles, P_1 and P_2 , on a coordinate line after t seconds are given by $s_1 = 3t^3 - 12t^2 + 18t + 5$ and $s_2 = -t^3 + 9t^2 - 12t$ respectively. When do the two particles have the same velocity?

Ans. _____

3. (15 p'ts.) $y = \frac{x^3+x^2}{2x^3}$. First find and simplify $\Delta y/\Delta x = \frac{f(x+\Delta x)-f(x)}{\Delta x}$. Then find dy/dx by taking the limit as $\Delta x \rightarrow 0$.

Ans. _____

4. (15 p'ts.) $x^2 + 4y^2 = 4$ describes an ellipse in the plane. Find all points on the ellipse where the tangent line has slope $2/3$.

Ans. _____

5. (10 p'ts.) Find the perpendicular bisector of the line segment connecting the points $(-2, 3)$ and $(2, 0)$.

Ans. _____

6. (10 p'ts.) Find the exact value of $\cos(\frac{7\pi}{12})$.

Ans. _____

7. (10 p'ts.) Find $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin(2\theta)}$.

Ans. _____

8. (10 p'ts.) Let $g(x) = \cos(\pi x) \sin^2(\pi x)$. Find $g'(1/2)$.

Ans. _____

PART II

1. (20 p'ts.) Find the volume of the solid generated by revolving the region bounded by the line $y = 3x$ and the parabola $y = 3x^2$ about the y - axis.

Ans. _____

2. (15 p'ts.) Evaluate the following:

a) $\int_1^2 (4x^3 + 7) dx$.

Ans. _____

b) $\int x \sin(x^2 + 4) dx$.

Ans. _____

c) $\int_{-\pi/2}^{\pi/2} z \sin^2(z^3) \cos(z^3) dz.$

Ans. _____

3. (10 p'ts.) $f(x) = 3x - 1$ on $[1, 4]$. Divide the interval $[1, 4]$ into three equal subintervals and calculate the area of the corresponding inscribed rectangular region.

Ans. _____

4. (20 p'ts.) $g(x) = 4x^3 - 3x^2 - 6x + 12$. Where is the graph of $g(x)$

a) increasing?

b) decreasing?

c) concave up?

d) concave down?

Ans. a) _____ b) _____

c) _____ d) _____

5. (20 p'ts.) A wire of length 100 centimeters is cut into two pieces; one piece is bent to form a square and the other is bent to form an equilateral triangle. Where should the cut be made if the sum of the two areas is to be a minimum?

Ans. _____

6. (15 p'ts.) Use the Mean Value Theorem to show that $f(x) = \tan(x)$ is an increasing function on $(-\pi/2, \pi/2)$.

Ans. _____