

**MATHEMATICS 221-CALCULUS I-FINAL**

**Tuesday, December 16, 2003**

**Instructor: Paul Milewski**

NAME (print): \_\_\_\_\_

**SECTION:** CARRACINO ISSA JOHNSON KANE SENKOWICZ SHI WU

1. Closed Book. Closed notes. No calculators.
2. Show all your work. Partial credit is given only if your work is clear.
3. Time allowed: 120 minutes. GOOD LUCK!

1. \_\_\_\_\_(20)
2. \_\_\_\_\_(20)
3. \_\_\_\_\_(20)
4. \_\_\_\_\_(20)
5. \_\_\_\_\_(20)
6. \_\_\_\_\_(20)
7. \_\_\_\_\_(20)
8. \_\_\_\_\_(20)
9. \_\_\_\_\_(20)
10. \_\_\_\_\_(20)
- Total. \_\_\_\_\_(200)

NAME (print): \_\_\_\_\_

1. Compute the following. Show the intermediate steps or justify your answers clearly.

(a)

$$\int x^{\frac{1}{3}} \sqrt{x^{\frac{4}{3}} + 7} dx$$

(b)

$$\int_{10}^{13} \frac{7}{2-x} dx$$

NAME (print): \_\_\_\_\_

(c) Find  $f'(x)$  for

$$f(x) = x^{\cos(x)}.$$

NAME (print): \_\_\_\_\_

2. (a) Find the equation of the tangent line to the curve

$$e^{xy} + \sin(xy) = 2 + x$$

at the point  $(-1,0)$ .

NAME (print): \_\_\_\_\_

(b) You are standing at your bedroom window, which is 15 feet off the ground. Your friend is outside walking directly towards you on the ground level at a constant speed of  $5 \frac{ft}{sec}$ . How fast is the distance between you and your friend changing when your friend is 20 feet from the building?

NAME (print): \_\_\_\_\_

3. Compute the following

(a)

$$\int_1^e \frac{\ln(x)}{x} dx$$

(b)

$$\int x^{-2} \sin(3x^{-1}) dx$$

NAME (print): \_\_\_\_\_

4. A goose, initially at  $50^\circ F$  is placed in an oven which is at a constant temperature of  $350^\circ F$ . One hour later the temperature in the middle of the goose is  $125^\circ F$ . Suppose the goose is cooked when its temperature reaches  $200^\circ F$ . Denote the temperature of the goose by  $\Theta(t)$  where  $t$  is time in hours. Assuming Newton's Law of heating and cooling,

$$\frac{d\Theta}{dt} = k(\Theta - \Theta_A).$$

(a) What is the sign of  $k$ ? What is  $\Theta_A$ ?

(b) What is the general solution to the equation?

(c) How long will it take the goose to cook?

NAME (print): \_\_\_\_\_

5. Calculate the following:

(a)

$$\lim_{t \rightarrow x} \frac{e^x - e^t}{x - t}$$

(b)

$$\lim_{x \rightarrow 0} x^{1/3} \ln(x)$$

NAME (print): \_\_\_\_\_

(c)

$$\lim_{h \rightarrow 0} \frac{1}{h} \int_x^{x+h} t^2 dt$$

If you did the problem right, you got a simple answer. Could you have guessed this answer without doing any calculations? Explain.

NAME (print): \_\_\_\_\_

6. Let  $y = f(x) = x^3 + x$ .

(a) Prove that  $f$  has an inverse. Hint: show that  $f$  has a simple property that guarantees an inverse ( $f^{-1}$ ) without actually calculating it.

(b) Find  $(f^{-1})'(2)$ .

NAME (print): \_\_\_\_\_

7. (a) Find a solution for

$$\frac{dy}{dx} = \frac{e^x y}{e^x + 1}$$

where  $y(0) = 17$ .

(b) Find the general solution for

$$\frac{dy}{dx} + \frac{2y}{x+2} = (x+2)^5$$

NAME (print): \_\_\_\_\_

8. Draw a continuous function on with the following properties:

$$f(-2) = 8, f(0) = 4, f(2) = 0,$$

$$f'(-2) = f'(2) = 0$$

$$f'(x) > 0 \text{ for } |x| > 2 \text{ and } f'(x) < 0 \text{ for } |x| < 2$$

$$f''(x) < 0 \text{ for } x < 0 \text{ and } f''(x) > 0 \text{ for } x > 0$$

NAME (print): \_\_\_\_\_

9. A simple model for tumor growth is that the volume of the tumor grows at a rate proportional to its surface area. Assume the tumor is a sphere of volume  $V(t)$ . The differential equation for  $V(t)$  is then

$$\frac{dV}{dt} = kV^\alpha.$$

Note that the area of a sphere is  $4\pi R^2$ .

(a) Why is  $\alpha = 2/3$ ? What is the sign of  $k$ ?

(b) Solve the differential equation.

(c) Suppose time is measured in months. Given that at  $V(0) = 1 \text{ cm}^3$  and  $V(1) = 8 \text{ cm}^3$ . When is the tumor  $1000 \text{ cm}^3$ ?

NAME (print): \_\_\_\_\_

10. Suppose the region bounded by the  $y$ -axis,  $x$ -axis,  $y = e^{-x}$  and  $x = a$  is revolved about the  $x$ -axis.

(a) What is the volume of the resulting "trumpet".

(b) Does the trumpet have a finite volume as  $a \rightarrow \infty$ ?

NAME (print): \_\_\_\_\_

SCRATCH WORK

NAME (print): \_\_\_\_\_

SCRATCH WORK