Math 221, Exam 1

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(1 point) Write your name _____

(1 point) Circle the name of your TA and time your class meets:

Ben Newton Jue Wang

8:50 9:55 11:00

- For full credit, SHOW YOUR WORK and CIRCLE YOUR ANSWERS.
- NO calculators, NO notes, NO looking at your neighbor's paper.

Problem	Points	Score
1	12	
2	12	
3	10	
4	8	
5	4	
6	15	
7	15	
8	8	
9	8	
10	8	
Totals	100	

1. **(12 Points)** Simplify the following expressions:

(a) $\log_{1/2}(8^{-x})$ **(b)** $e^{3\ \ln(x)}$ **(c)** $2^{5\log_2(x)}$ **(d)** $\ln(x^2-1)-\ln(x+1)$

2. **(12 Points)** Calculate the equations for the following lines:

(a) the line passing through (2,4) with slope $-\frac{1}{2}$

(b) the vertical line through (2,-3)

(c) the line passing through (2,-3) and parallel to $x + 2y - 4 = 0$

(d) the line passing through (4,-1) and perpendicular to the line passing through (-2,0) and (1,1)

3. **(10 Points)** Use the $\epsilon - \delta$ definition of the limit to prove $\lim_{x \to 1}(3x - 1) = 2$

4. **(8 Points)** Find the exponential growth equation (ie: an explicit formula for N_t) or recursion formula (ie: a formula for $N_{t+1} = f(N_t)$ and N_0)

(a) Find the exponential growth equation for a population that doubles in size every unit of time and that has 53 individuals at time 0.

(b) Find the recursion formula for a population that triples in size every unit of time and has 10 individuals at time 0.

5. **(4 Points)** Calculate

(a) $\lim_{t \to 0} \frac{\sin^2(3t)}{2t}$

(b) $\lim_{x \to 0} \frac{x+1-\cos(2x+1)}{x}$

6. **(15 Points)** Calculate the following limits, or explain why they don't exist. Justify your answers.

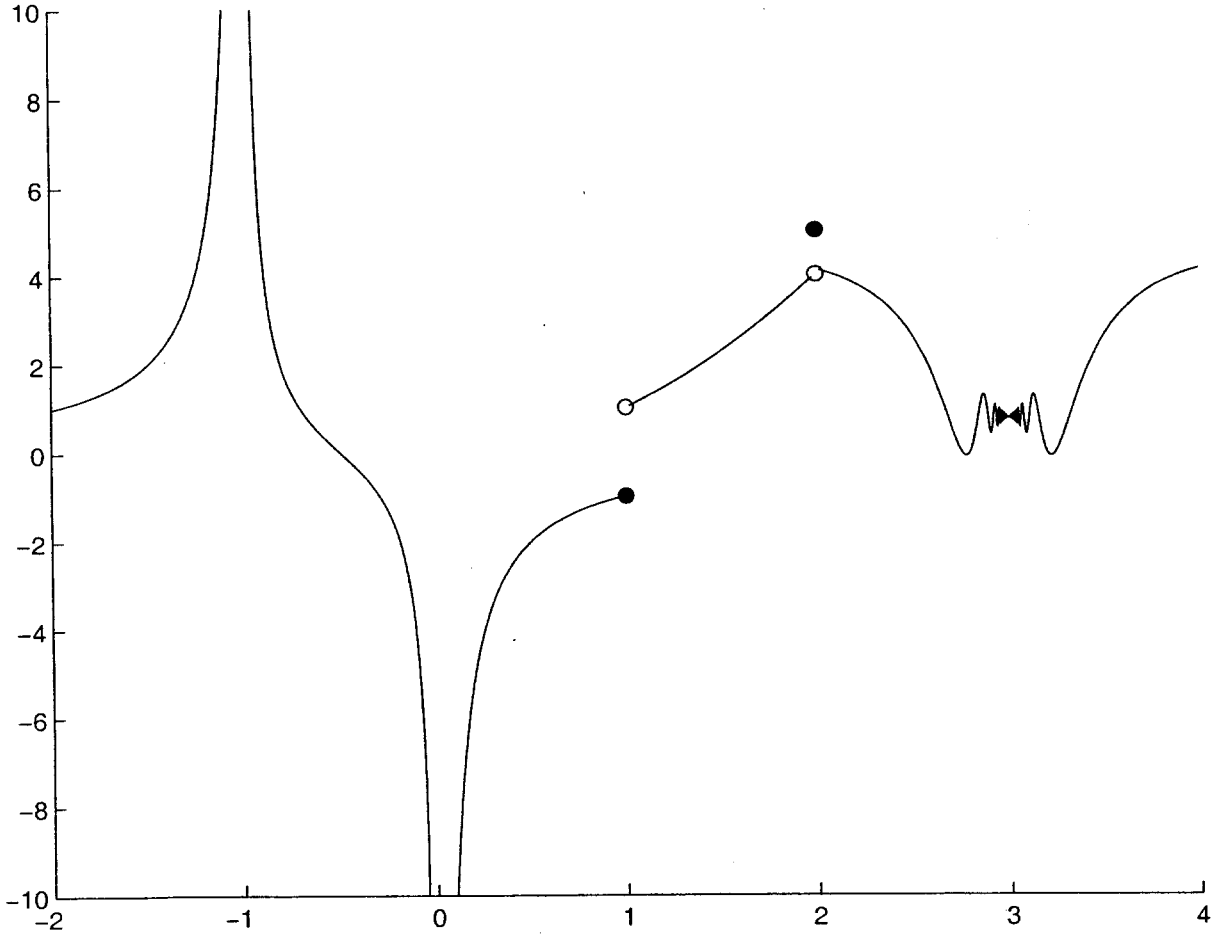
   (a) $\lim_{x \to 2} \frac{x^2-4}{x+2}$

   (b) $\lim_{x \to -\infty} e^x$

   (c) $\lim_{x \to \infty} \frac{1-x^2}{x^2}$

7. **(15 Points)** Use the graph of $f(x)$ to determine the properties of f at the points given. For each point, write "Yes" or "No" in the box to indicate whether $f(x)$ has the property at the point $x = c$. (Note that vertical asymptotes exist at $x = -1, 0$.)

Point	$\lim_{x \to c} f(x)$ exists $(\neq \infty, -\infty)$	$\lim_{x \to c^+} f(x) = \infty$	Continuous
$c = -1$			
$c = 0$			
$c = 1$			
$c = 2$			
$c = 3$			



8. **(8 Points)**

(a) Define $f(x) = \frac{x^3-1}{x-1}$ for $x \neq 1$. Can we define $f(1)$ so that the function f is continuous at $x = 1$? Justify your answer.

(b) Repeat this question for $g(x) = \ln(|x-1|)$.

9. **(8 Points)** Write a explicit formula for a_n, where $\{a_n\}$ is defined by the following sequence. Also, calculate a_0, a_1, a_2 explicitly to check your formula.

(a) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}, \ldots$

(b) $0, \frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \ldots$

10. **(8 Points)** Let N_t be defined by the recursive relationship

$$N_{t+1} = \frac{1}{2}(N_t + N_{t-1})$$

(a) If $N_0 = 0$ and $N_1 = 1$, calculate N_2, N_3 and N_4.
(b) Derive an explicit formula for N_t.
(c) What is $\lim_{t\to\infty} N_t$? Justify your answer.