

# CALCULUS 222

## 11th WEEK EXAM

I. M. Isaacs  
Thursday, April 12, ~~2006~~ 2007  
5:30 - 7:00 P.M.

Do all problems — 100 points.  
Use backs of pages for scrap, or if you need more space.

NAME: \_\_\_\_\_

TA: \_\_\_\_\_

Do not write below here.

Prob. 1: \_\_\_\_\_ out of 25.

Prob. 2: \_\_\_\_\_ out of 10.

Prob. 3: \_\_\_\_\_ out of 20.

Prob. 4: \_\_\_\_\_ out of 20.

Prob. 5: \_\_\_\_\_ out of 15.

Prob. 6: \_\_\_\_\_ out of 10.

Total: \_\_\_\_\_ out of 100.

1. [25 POINTS] Find all of the following for the ellipse  $25x^2 + 16y^2 = 400$ .

Length of major axis: \_\_\_\_\_

Length of minor axis: \_\_\_\_\_

Coordinates of foci: ( \_\_\_\_\_ , \_\_\_\_\_ ) and ( \_\_\_\_\_ , \_\_\_\_\_ )

Equations of directrices: \_\_\_\_\_ and \_\_\_\_\_

Eccentricity: \_\_\_\_\_

The total length of the path described below: \_\_\_\_\_

Start at one focus, move up and to the right along a line having slope 1 until the ellipse is reached at some point  $P$ , and then and then move along a straight line from  $P$  to the other focus.

Now a new ellipse is drawn. Its polar coordinate equation is  $r = \frac{A}{1 + B \cos \theta}$ .

Find positive numbers  $A$  and  $B$  such that the new ellipse has exactly the same size and shape of the original ellipse.

$A =$  \_\_\_\_\_ and  $B =$  \_\_\_\_\_

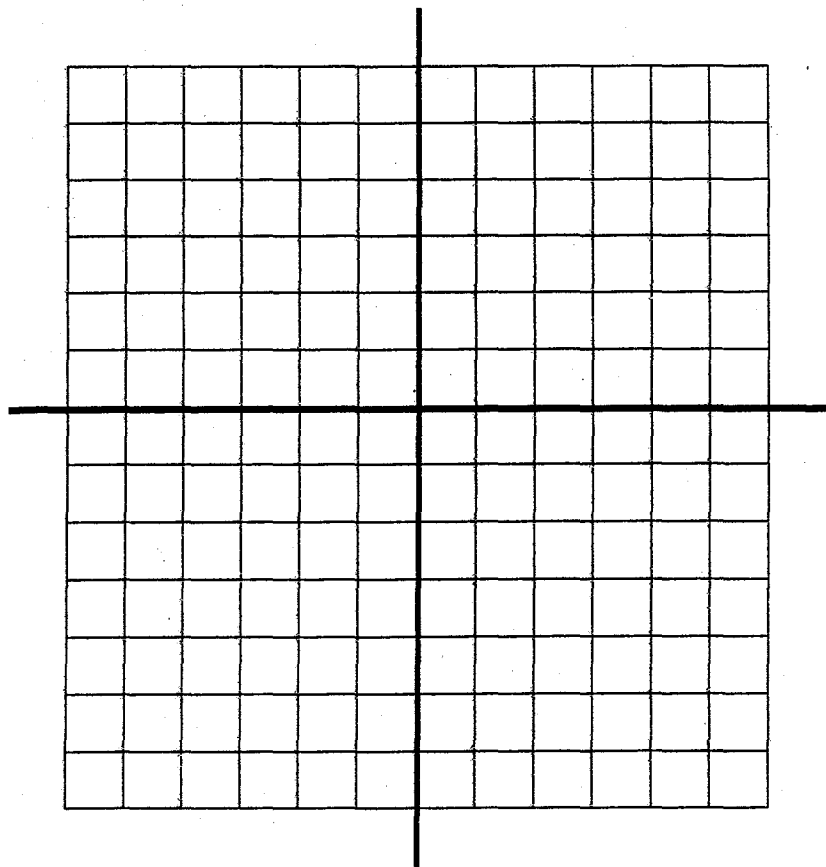
Finally, find the Cartesian (that is  $x, y$ ) coordinates of the foci of the new ellipse.

Coordinates of foci: ( \_\_\_\_\_ , \_\_\_\_\_ ) and ( \_\_\_\_\_ , \_\_\_\_\_ )

2. [10 POINTS] Draw a reasonably accurate sketch of the graph of

$$4x^2 - 8x - 9y^2 + 36y = 68.$$

Draw the asymptotes and mark the foci. (NEATNESS COUNTS ON THIS PROBLEM.)



3. [20 POINTS]

(a) The graph of the polar coordinate equation  $r = 2(\sin(\theta) + \cos(\theta))$  is a circle. Find its center and radius.

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(b) Compute the area that is common to the interiors of the circle of part (a) and the circle whose polar coordinate equation is  $r = 2 \cos(\theta)$ .

4. [20 POINTS] Let  $f(x) = \frac{\sin(x^2)}{1+x^3}$ .

(a) Compute the Maclaurin series of  $f(x)$  up to and including the  $x^9$  term.

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(b) Compute the value of the ninth derivative of  $f(x)$  at  $x = 0$ .

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(c) Compute  $\lim_{x \rightarrow 0} \frac{f(x) - x^2}{x^5}$ .

5. [15 POINTS] Let  $f(x) = \sin\left(\frac{\pi}{2}x\right)$ . Find the Taylor series for the function  $f(x)$  expanded about the point  $x = 1$ . (Write several terms, and then write the whole series in  $\Sigma$ -notation.)

6. [10 POINTS] I have a function  $f(x)$  that can be differentiated as many times as I want, and I can prove that for every number  $k \geq 0$ , the  $k$ th derivative  $f^{(k)}$  of  $f$  satisfies the inequality  $|f^{(k)}(x)| < k!$  for all numbers  $x$  such that  $-2 < x < 2$ . Let  $P_n(x)$  be the  $n$ th Taylor polynomial for  $f(x)$  expanded around  $x = 0$ . I want to use these Taylor polynomials to approximate the function  $f(x)$ .

(a) I am sure that if I take  $n$  to be large enough, I can guarantee that the absolute value of the difference between  $P_n(.1)$  and  $f(.1)$  is very small. In other words, I know that  $\lim_{n \rightarrow \infty} P_n(.1) = f(.1)$ . Explain why this is true.

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(b) Suppose I want to be sure that the absolute value of the difference between  $P_n(.1)$  and  $f(.1)$  is less than  $10^{-6}$ . How large should I take  $n$ ?

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(c) Suppose that I want to approximate  $f(1.1)$  by  $P_n(1.1)$ . Can I be sure that if  $n$  is very large, this will be a good approximation? Explain.

THE END