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CALCULUS 222

FINAL EXAM

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Thursday, May 17, 2007
10:05 A.M. – 12:05 P.M.

Do all problems — 200 points.
Use backs of pages for scrap, or if you need more space.

NAME: _____

TA: _____

Do not write below here.

Prob. 1: _____ out of 30.

Prob. 2: _____ out of 25.

Prob. 3: _____ out of 30.

Prob. 4: _____ out of 27.

Prob. 5: _____ out of 25.

Prob. 6: _____ out of 33.

Prob. 7: _____ out of 30.

Total: _____ out of 200.

1. [30 POINTS] Integrate:

(a) $\int \frac{\ln(x)}{x^2} dx$

(b) $\int_{\sqrt{2}}^{\sqrt{3}} \frac{x^2 dx}{(4-x^2)^{3/2}}$

2. [25 POINTS]

(a) In order to approximate $\sqrt{0.9}$ without a calculator, I decide to use the Taylor series for the function $f(x) = \sqrt{x}$, expanded about the point $x = a$. What would be a good choice for a ? Using the value for a that you selected, and using the quadratic (in other words, degree 2) Taylor polynomial, what approximate value for $\sqrt{0.9}$ is obtained?

(b) Find the worst possible error in this approximation according to the Taylor remainder error estimate.

3. [30 POINTS] Solve these initial value problems.

(a) $xy' - 2y = 2x^4$ $y = 3$ when $x = 1$.

(b) $y'' - 6y' + 9y = 27x$ $y = 3$ and $y' = 10$ when $x = 0$.

4. [27 POINTS] An ellipse with foci at $(0, 0)$ and $(-4, 0)$ crosses the y -axis at the point $(0, 3)$. Find a polar coordinate equation and a Cartesian coordinate equation for the ellipse. Also, where does the ellipse cross the x -axis?

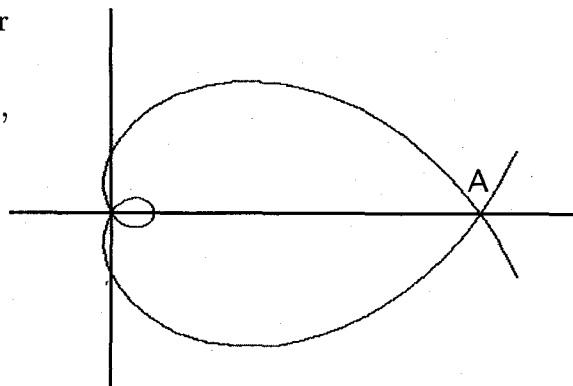
(a) Polar coordinate equation: _____

(b) Cartesian coordinate equation: _____

(c) Crosses x -axis at: $x =$ _____ and $x =$ _____

5. [25 POINTS] Shown here is part of the polar coordinate graph of $r = 1 - \theta^2$.

(a) Find the Cartesian coordinates of the point A , where the curve crosses itself and the x -axis.



(b) Find the area of the little loop to the right of the origin.

(c) Suppose that a particle travels along this curve so that $\theta = 2t$ at time t . Find the velocity and acceleration vectors (expressed in terms of \vec{i} and \vec{j}) when $t = 0$. **HINT:** First, express the velocity and acceleration in terms of \vec{u}_r and \vec{u}_θ .

6. [33 POINTS] Given points $A:(-1, 1, 4)$, $B:(0, 2, 2)$ and $C:(2, 1, 1)$, find the following.

(a) Angle BAC .

(b) The equation of the plane containing points A , B and C .

(c) The point where the line through B perpendicular to line AC meets AC .

7. [30 POINTS] A particle moves in space: $x = e^t$, $y = t^2$ and $z = t$, where t is time.
- (a) At time $t = 0$, find: the velocity, the speed, the acceleration and the unit tangent vector \vec{T} .

(b) The acceleration \vec{a} can be written in the form $\vec{a} = c\vec{T} + d\vec{N}$, where c and d are scalars and \vec{N} is the unit normal vector. Find c and d at time $t = 0$. **HINT:** Find c first, using a dot product.

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- (c) At time $t = 0$, find the curvature and the rate of change of speed.
HINT: Use the scalars c and d from the previous part.

Have a good summer vacation.