MATH 222 — THIRD MIDTERM

April 10, 2002, 9:55am–10:45am

Your Name:

Your TA: (circle one)

Chris Alfeld **Graham Jonaitis** **Andy Raich**

Joshua Rushton **Fernando Miranda**

Score

1: 2: 3 or 6: 4: 5:

Total:

THIS EXAM HAS SIX PROBLEMS
YOU SHOULD ONLY DO FIVE OF THESE
EVERYONE MUST DO PROBLEMS 1,2,4,5
YOU MUST CHOOSE BETWEEN PROBLEMS 3 AND 6
DO EITHER 3 OR 6 BUT NOT BOTH

(1) Find the solution of the differential equation

$$\frac{dy}{dx} = (1 + y^2) \sin x,$$

which satisfies $y(\frac{\pi}{2}) = 3$.

(2) Find the solution of the differential equation

$$x\frac{dy}{dx} + (1-x)y = 1 - x$$

which satisfies $y(1) = 0$.

Do either this problem or problem 6 (on polar coordinates) but not both

(3) In the problems on this page x is an abitrary real number.

(a) Compute $z = \dfrac{x+i}{7+i}$ (i.e. write z as $a+bi$ with a and b real.)

(b) Assuming $x > 0$, draw $x+ix\sqrt{3}$ and compute $\arg(x+ix\sqrt{3})$

(c) Assume $0 < x < \pi/2$.
Draw e^{ix}, e^{2ix} and $e^{ix} + e^{2ix}$ in one figure.
Then compute $z = e^{ix} + e^{2ix}$, i.e. rewrite $z = a + bi$ with a and b real.

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(4) Find the general solution of

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + \alpha y = 0,$$

where α is some positive constant. Your answer should be valid for all possible choices of $\alpha > 0$.

(5) Find the general solution of

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t.$$

Do either this problem or problem 3 (on complex numbers) but not both

(6) (a) Find the equation in Cartesian Coordinates for the curve which in Polar Coordinates is given by $r^2 = \sin\theta\cos\theta$.

(b) Find the equation in Polar Coordinates for the curve which in Cartesian Coordinates is given by $x^4 + y^2 = 3x$.

(c) Consider a Logarithmic Spiral which in Polar Coordinates is given by $r = e^{\theta/2\pi}$. For which value(s) of θ between 0 and 2π does this spiral have a horizontal tangent?

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