

Midterm Exam 1, Friday, Oct 7, 2005
Lev Borisov, Math 222, Lecture Section 1

**DO NOT OPEN THE EXAM
BEFORE THE START
ANNOUNCEMENT !**

Please write your name and your TA's name below.

Name:

TA:

Each problem is worth 20 points, for a total of 100 points. **Calculators are not allowed on this test.** Please read each question carefully, it also helps to check afterwards that you have answered each part of each question. **You must show all your work to receive credit.** When you turn in your paper after the test, make sure the TA checks your name in their list or writes your name down. Good luck!

1	2	3	4	5	Total

[1] (a)[10pts] Give an example of a divergent sequence $\{a_n\}$ such that the sequence $\{a_n^2\}$ converges.

(b)[10pts] Give an example of a convergent series $\sum_{n=1}^{\infty} b_n$ such that the series $\sum_{n=1}^{\infty} b_n^2$ diverges.

You must justify your answers. *Hint: this may well be the hardest problem of this test. If you can't solve it after a few minutes, move on to other problems and then come back to this one, if there is time left.*

[1] (20 pts)

Please leave blank!

[2] For each of the following series determine if it converges absolutely, converges conditionally or diverges.

(a)[6pts] $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

(b)[7pts] $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n + 2 \ln n}$

(c)[7pts] $\sum_{n=1}^{\infty} \frac{(-1)^n (n!)^3}{(3n)!}$

[2] (20 pts)

Please leave blank!

[3] Determine for which x the power series

$$\sum_{n=0}^{\infty} \frac{3^n}{n^2 - 2} x^n$$

converges absolutely, converges conditionally, diverges.

[3] (20 pts)

Please leave blank!

[4] Find the sum of the following convergent series. You must justify your answers.

(a)[6pts] $\sum_{n=1}^{\infty} \frac{5}{4^n}$

(b)[7pts] $\sum_{n=1}^{\infty} \frac{(-1)^n \pi^{2n}}{4^n (2n)!}$

(c)[7pts] $\frac{1}{2!} + \frac{2}{3!} + \frac{2^2}{4!} + \dots + \frac{2^n}{(n+2)!} + \dots$

[4] (20 pts)

Please leave blank!

[5] Calculate the Maclaurin series up to x^2 of the function $f(x) = e^x \ln(1+x)$ in two different ways. First, use the definition of the Maclaurin series in terms of the derivatives at zero. Second, use the known Maclaurin series for e^x and $\ln(1+x)$. Each part is worth 10 points.

[5] (20 pts)

Please leave blank!