

Midterm Exam 1, Friday, Oct 7, 2005
Lev Borisov, Math 222, Lecture Section 3

**DO NOT OPEN THE EXAM
BEFORE THE START
ANNOUNCEMENT !**

Please write your name and your TA's name below.

Name:

TA:

Each problem is worth 20 points, for a total of 100 points. **Calculators are not allowed on this test.** Please read each question carefully, it also helps to check afterwards that you have answered each part of each question. **You must show all your work to receive credit.** When you turn in your paper after the test, make sure the TA checks your name in their list or writes your name down. Good luck!

1	2	3	4	5	Total

[1] The sequence $\{a_n\}$ is given by $a_1 = 1$ and the recursive relation

$$a_{n+1} = a_n \cdot 2^{(1/3)^n}.$$

This means that $a_2 = \sqrt[3]{2}$, $a_3 = \sqrt[3]{2}\sqrt[3]{2}$, $a_4 = \sqrt[3]{2}\sqrt[3]{2}\sqrt[3]{2}$, and so on. Find the limit

$$\lim_{n \rightarrow +\infty} a_n$$

or show that it does not exist.

[1] (20 pts)

Please leave blank!

[2] For each of the following series determine if it converges absolutely, converges conditionally or diverges.

(a)[7pts] $\sum_{n=1}^{\infty} \frac{2^n n!}{e^{n^2}}$

(b)[7pts] $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n \ln n}$

(c)[6pts] $\sum_{n=1}^{\infty} (\sqrt{n^2 + 1} - n)$

[2] (20 pts)

Please leave blank!

[3] Determine for which x the power series

$$\sum_{n=0}^{\infty} \frac{1}{(n+3)} (2x+1)^n$$

converges absolutely, converges conditionally, diverges.

[3] (20 pts)

Please leave blank!

[4] Find the sum of the following convergent series. You must justify your answers.

(a)[6pts] $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{4^{2n+1}(2n+1)!}$

(b)[14pts] $\sum_{n=0}^{\infty} (n+1) \left(\frac{\pi}{4}\right)^n$

[4] (20 pts)

Please leave blank!

- [5] (a)[5pts] State the remainder formula for the Taylor polynomial.
(b)[15pts] How many terms of the series

$$\sum_{n=0}^{\infty} \frac{(1/2)^n}{n!}$$

does one need to calculate to get within $\frac{1}{1000}$ of its sum \sqrt{e} . You can use here the inequality $\sqrt{e} < 2$.

[5] (20 pts)

Please leave blank!