Math 234 | Fall 2007

Exam I | S. Bolotin

Your Name: ______________________________

Your TA: ______________________________

Your Section Meeting Time: ______________________

PROBLEM	POINTS	SCORE
I	20	
II	20	
III	20	
IV	20	
V	20	
TOTAL	100	

No calculators, texts or notes. Show all your work: no work – no credit. Circle your answer. You may use the last page as scratch paper, but it will not be checked.

I. (20 points) (a) Find the directional derivative of the function $f(x, y, z) = xy + \ln(3x + y^2) + z^2$ at the point $P(-1, 2, 2)$ in the direction towards the origin.

(b) Find the equation of the plane tangent to the surface $f(x, y, z) = 2$ at the point $P(-1, 2, 2)$.

(c) Equation $f(x, y, z) = 2$ defines $z = z(x, y)$ as a function of x, y. Find $\frac{\partial z}{\partial y}$ at $P(-1, 2, 2)$.

II. (20 points) Let $z = x^2 e^{y-1}$ with $x = s^2 + t^2$ and $y = st$.

(a) Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ at $(s, t) = (1, 1)$.

(b) Use linear approximation to estimate the value of z when $s = 1.1$ and $t = 0.9$.

III. (20 points) (a) Find all critical points of the function $f(x, y) = x^3 + y^3 + 3xy$.

(b) Identify the critical points as local minima, maxima, or saddle points.

IV. (20 points) Let $\mathbf{r}(t) = e^t\mathbf{i} - e^{-t}\mathbf{j} + (t\sqrt{2}+1)\mathbf{k}$ be the position vector of a moving particle. Find: (a) The arc length traveled for $0 \le t \le 1$.

(b) Unit tangent vector $\mathbf{T}$ to the path at the point $P(1,-1,1)$.

(c) Unit principal normal vector $\mathbf{N}$ to the path at $P(1,-1,1)$.

V. (20 points) Find the maximal and minimal values of the function $f(x, y, z) = x - y + 3z$ subject to the constraint $x^2 + y^2 + 4z^2 = 4$.