Math 234 | Fall 2007

Final Exam | S. Bolotin

Your Name: ____________________

Your TA: ____________________

Your Section Meeting Time: ____________________

PROBLEM	POINTS	SCORE
I	25	
II	25	
III	25	
IV	25	
V	25	
VI	25	
VII	25	
VIII	25	
TOTAL	200	

Show all your work: no work – no credit. Hand in your formula sheet together with the exam to your TA.

I. (25 points) Let $f(x, y, z) = xe^z + (\ln y)(\sin z) - y$.

(a) In which direction is f increasing most rapidly at the point $(2, 1, 0)$? Express the direction as a unit vector $\mathbf{u}$.

ANSWER: $\mathbf{u} =$

(b) Find an equation of the plane through the point $(2, 1, 0)$ which is tangent to the level surface of f passing through $(2, 1, 0)$.

ANSWER:

(c) Use linear approximation to find approximately the value of $f(x, y, z)$ at $(2.01, 0.9, 0.1)$.

ANSWER: $f(2.01, 0.9, 0.1) \cong$

II. (25 points) Let $f(x, y) = x^2 + xy + y^2 - 6x$. (a) Find the critical points of $f(x, y)$ and identify them as local maxima, minima or saddle points.

ANSWER:

(b) Find the global maximum and minimum values of $f(x, y)$ in the closed rectangle $0 \le x \le 5$, $-3 \le y \le 3$.

ANSWER: max= min=

III. (25 points) Find $\iiint_D y\,dV$, where D is the solid tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$.

ANSWER: $\iiint_D y\,dV =$

IV. (25 points) Let $\mathbf{F} = (e^x \cos y + yz)\mathbf{i} + (xz - e^x \sin y)\mathbf{j} + xy\mathbf{k}$.

(a) Is $\mathbf{F}$ conservative? Justify your answer.

ANSWER:

(b) Find a function f such that $\mathbf{F} = \nabla f$. Justify your answer.

ANSWER: $f(x, y, z) =$

V. (25 points) The mass density of the hemispherical shell $x^2 + y^2 + z^2 = 1$, $z \geq 0$ is given by $\delta(x, y, z) = z$. (a) Find the mass M of the shell.

ANSWER: $M =$

(b) Find the center of mass $(\bar{x}, \bar{y}, \bar{z})$ of the shell.

ANSWER: $\bar{x} =$ $\bar{y} =$ $\bar{z} =$

VI. (25 points) Show that the integral $\int_C (2xy - y)\,dx + x^2\,dy$ along a simple closed curve C in the plane equals the area of the region R bounded by C. The curve is oriented counter clock wise.

VII. (25 points) Let D be the solid cylinder $0 \leq x^2 + y^2 \leq 9$, $0 \leq z \leq 5$. Let S be the boundary of the cylinder. Find the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma$, where $\mathbf{F}(x, y, z) = (x - e^y)\mathbf{i} + (y - \cos z)\mathbf{j} + z^2\mathbf{k}$, and $\mathbf{n}$ is the outer normal to S.

ANSWER: $\iint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma =$

VIII. (25 points) Let S be the surface of the hemisphere $z = \sqrt{4 - x^2 - y^2}$, $0 \leq x^2 + y^2 \leq 4$. Find the surface integral $\iint_S (\text{curl}\,\mathbf{F}) \cdot \mathbf{n}\, d\sigma$, where $\mathbf{F}(x, y, z) = (y + e^{z^2} - 1)\mathbf{i} + (\sin^2 z - x)\mathbf{j} + z^2\mathbf{k}$.

ANSWER: $\iint_S (\text{curl}\,\mathbf{F}) \cdot \mathbf{n}\, d\sigma =$