

Midterm Exam 1, Monday, Oct 8, 2007

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**DO NOT OPEN THE EXAM  
BEFORE THE START  
ANNOUNCEMENT !**

Please write your name and your TA's name below.

**Name:**

**TA:**

Each problem is worth 20 points, for a total of 100 points. **Calculators are not allowed on this test.** Please read each question carefully, it also helps to check afterwards that you have answered each part of each question. **You must show all your work to receive credit.** When you turn in your paper after the test, make sure the TA checks your name in their list or writes your name down. Good luck!

1	2	3	4	5	Total

[1] Find the curvature of the parametric curve given by

$$(x(t), y(t), z(t)) = (t, t^2, 2t + 1)$$

at the point  $P(2, 4, 5)$ .

[1] (20 pts)

Please leave blank!

[2] Find the limit of the function of two variables or show that it does not exist.

(a)[10pts]  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 - y^2}{x^2 + y^2}$

(b)[10pts]  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2 + 1}$

[2] (20 pts)
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Please leave blank!

[3] Find the direction of maximum increase of the function

$$f(x, y, z) = yx^2 + z^2$$

at the point with coordinates  $(-1, 2, -2)$ . Find the directional derivative of the function  $f$  in this direction.

[3] (20 pts)

Please leave blank!

[4] (a)[10pts] Calculate the tangent plane to the sphere  $x^2 + y^2 + z^2 = 1$  at the point with coordinates  $(a, b, c)$ .

(b)[10pts] Find all tangent planes to the sphere  $x^2 + y^2 + z^2 = 1$  which are parallel to the plane  $2x - y + 2z = 2007$ .

[4] (20 pts)
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Please leave blank!

[5] (a)[6pts] State the second derivative test for functions of two variables.

(b)[7pts] Give an example of a function  $f(x, y)$  with a local minimum at  $(x, y) = (0, 0)$ . You must justify your answer.

(c)[7pts] Give an example of a function  $f(x, y)$  with a local saddle point at  $(0, 0)$ . You must justify your answer.

[5] (20 pts)

Please leave blank!