Midterm Exam 2, Monday, Nov. 12, 2007

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DO NOT OPEN THE EXAM BEFORE THE START ANNOUNCEMENT !

Please write your name and your TA's name below.

Name: **TA:**

Each problem is worth 20 points, for a total of 100 points. **Calculators are not allowed on this test.** Please read each question carefully, it also helps to check afterwards that you have answered each part of each question. **You must show all your work to receive credit.** When you turn in your paper after the test, make sure the TA checks your name in their list or writes your name down. Good luck!

1	2	3	4	5	Total

These are the formulas from section 15.5. You have to know how to write similar formulas for line integrals and double integrals. Here $\delta = \delta(x, y, z)$ is a mass density function for a domain D in space. All triple integrals are over the domain D.

Mass and center of mass:

$$M = \iiint \delta \, dV, \quad \bar{x} = \frac{1}{M} \iiint x\delta \, dV, \quad \bar{y} = \frac{1}{M} \iiint y\delta \, dV, \quad \bar{z} = \frac{1}{M} \iiint z\delta \, dV$$

Moments of inertia about the coordinate axes:

$$I_x = \iiint (y^2 + z^2)\delta \, dV, \quad I_y = \iiint (x^2 + z^2)\delta \, dV, \quad I_z = \iiint (x^2 + y^2)\delta \, dV$$

Radii of gyration about the axes:

$$R_x = \sqrt{\frac{I_x}{M}}, \quad R_y = \sqrt{\frac{I_y}{M}}, \quad R_z = \sqrt{\frac{I_z}{M}}$$

[1] Sketch the region of integration and calculate the integral

$$\int_{y=0}^{1/16} \int_{x=y^{1/4}}^{1/2} \cos(16\pi x^5)\, dx dy.$$

Hint: The antiderivative $\int \cos(16\pi x^5)\, dx$ can not be expressed in terms of standard functions.

[1] (20 pts)

Please leave blank!

[2] Find the centroid of the quarter of the circle

$$x^2 + y^2 \leq 4, \; x \geq 0, \; y \geq 0.$$

Remark: By symmetry, the centroid will lie on the line $y = x$.

[2] (20 pts)

Please leave blank!

[3] Sketch the region of integration and calculate the integral

$$\int_{x=-1}^{1} \int_{y=0}^{\sqrt{1-x^2}} \int_{z=-\sqrt{1-x^2-y^2}}^{0} \sqrt{x^2+y^2+z^2}\; dzdydx.$$

[3] (20 pts)

Please leave blank!

[4] The curve C in space is the intersection of the cylinder $x^2 + y^2 = 1$ with the plane $x + y + z = 10$ oriented counterclockwise as viewed from above.

(a)[10pts] Write a parametric equation of C.

(b)[10pts] Calculate the work over C of the vector field $\vec{F}(x, y, z) = \overrightarrow{(y^{-1}, 0, 2)}$.

[4] (20 pts)

Please leave blank!

[5] Calculate

$$\int_C (3x + y^2)\, dx + \cos(e^y + 1)\, dy$$

where C is the boundary of the triangle with vertices $(1, -1)$, $(0, 0)$ and $(0, 2)$, oriented clockwise.

[5] (20 pts)

Please leave blank!