Math 234 Calculus of Several Variables
Final Exam
Instructor: Jun Chen

No calculator allowed.
Write detailed work to obtain full credits.

Your Name:

Please circle your TA session:

8:50T 9:55T 8:50R 8:55R

11:00T 12:05T 11:00R 12:05R

Problem/pt	Score
P1/30	
P2/35	
P3/35	
P4/35	
P5/30	
P6/35	
Total/200	

1. (30 pt) Evaluate the following double integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \cos(x^2 + y^2)\, dy\, dx.$$

2. (35 pt) The shape of an ice cream is part of the unit ball: $x^2+y^2+z^2 \leq 1$ inside the upper part of the cone: $z = \sqrt{x^2+y^2}$. The density function $\delta(x, y, x) = z$.

Set up the integrals for the following quantities, but **do not evaluate!**

(a) the mass of the ice cream (b) center of the mass.

(Hint: use spherical coordinates and some symmetry property.)

3. (35 pt) Let D be the domain in the first quadrant of x-y plane bounded by the following four curves:

$$y = \sqrt{x}, \quad y = 2\sqrt{x}, \quad y = \frac{1}{\sqrt{x}}, \quad y = \frac{2}{\sqrt{x}}.$$

Find the area of domain D (Hint: Change variables $u = \frac{y}{\sqrt{x}}, v = \sqrt{x}\,y$).

4. (35 pt) Determine whether the vector field

$$\mathbf{F} = (y\cos(xy) + 2xy^3, x\cos(xy) + 3x^2y^2)$$

is conservative or not. If it is conservative, find its potential function.

6. (35 pt) Let $\mathbf{F} = (x^2y, yz, x)$. Let G be the upper hemisphere $z = \sqrt{1-x^2-y^2}$ and $\mathbf{n}$ be the unit normal pointing upward.

(a) Use Stokes' theorem to identify the surface integral

$$\iint_G \nabla \times \mathbf{F} \cdot \mathbf{n}\, dS.$$

with a line integral.

(b) Use Stokes' theorem again to evaluate the above surface integral by a double integral in x-y domain.