Your name:_____

TA(circle one): Matt Davis Hongnian Huang Boian Popunkiov

Math 234 **Lecture 3(Feldman)**
May 10, 2006 **Final Exam**

Write your answers in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answer in exact forms (using $\frac{\pi}{2}$, $\sqrt{3}$, $\cos 1.2$, e^2, etc.)

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
5	25	
6	25	
7	25	
8	25	
TOTAL	200	

1. (25 points) Let a surface be given by $z = x^3 - xy + y^2 - x$.

(a) Find all points (x, y, z) on the given surface where the tangent plane is horizontal.

(b) Find the equation of tangent plane to the given surface at the point $(1, 2, 4)$.

2. (25 points) A curve is given by

$$\mathbf{r}(t) = e^t\mathbf{i} + te^t\mathbf{j} + t^2\mathbf{k}.$$

(a) Find the curvature, the unit tangent vector, the principal normal, and the binormal of the given curve $\mathbf{r}(t)$ at $t = 0$.

(b) Find the parametric equations of the tangent line to the given curve $\mathbf{r}(t)$ at $t = 0$.

3. (25 points) Find all critical points of the function $f(x,y) = x^2y + \frac{4}{x} + \frac{4}{y}$ and indicate for each such point whether it gives a local minimum or a local maximum, or whether it is a saddle point.

4. (25 points) Find the maximum and minimum values of $f(x, y) = 2x - y$ subject to the constraint $x^2 + y^2 - xy = 1$, and indicate where each occurs.

5. (25 points) Find the mass of the solid region inside the sphere of radius 2 and outside the sphere of radius 1 (both spheres are centered in the origin) and above the plane $z = 0$, if the density is $\delta(x, y, z) = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}}$.

6. (25 points) Let G be the part of surface $z = 1 - y^2$ above the rectangle $0 \leq x \leq 2$, $0 \leq y \leq 1$. Find

$$\iint_G xy \, dS.$$

7. (25 points) Let $\mathbf{F} = y\mathbf{i} + xz\mathbf{j} + e^z\mathbf{k}$. Calculate the flux of $\mathbf{F}$ through the boundary of the solid region S defined by $1 \leq x^2 + y^2 \leq 9$, $1 \leq z \leq 3$, that is calculate

$$\iint_{\partial S} \mathbf{F} \cdot \mathbf{n}\, dS.$$

Hint. You may use Gauss' divergence theorem.

8. (25 points) Calculate

$$\oint_C e^x\,dx + 2xy\,dy,$$

where C is the ellipse $9x^2 + y^2 = 9$.
Hint. You may use Green's theorem.