

**MATHEMATICS 234—CALCULUS III—MIDTERM I**

**Tuesday, October 10, 2006**

**Instructor: Paul Milewski**

NAME (print): \_\_\_\_\_

**Instructions:**

1. Write your name on each page.
2. Circle the name of your TA from the list below.

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3. Closed Book. No calculators. 1 sheet notes allowed.
4. Answer your questions on the exam paper. There are some extra pages in the back if you need them.
5. Show all your work. Partial credit is given only if your work is clear.
6. Time allowed: 75 minutes. GOOD LUCK!

1. \_\_\_\_\_(20)

2. \_\_\_\_\_(20)

3. \_\_\_\_\_(20)

4. \_\_\_\_\_(20)

5. \_\_\_\_\_(20)

Total. \_\_\_\_\_(100)

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1) Consider the function

$$f(x, y) = -x^2 - \frac{1}{4}y^2$$

(a) In the  $xy$  plane sketch *carefully* the level curves of the function for  $f = 0$ ,  $f = -1$ ,  $f = -4$ .

(b) Sketch the quadric surface  $x^2 + \frac{1}{4}y^2 + z = 0$ .

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(c) What is the relation between your sketch for (a) and the sketch for (b)?

(d) Write the equation for the quadric surface of part (b) in spherical coordinates in the form  $\rho = g(\theta, \phi)$ .

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2) If the **velocity** of a moving particle is given by

$$\mathbf{v}(t) = e^t \cos(t) \mathbf{i} + e^t \sin(t) \mathbf{j} + \sqrt{3}e^t \mathbf{k}.$$

(a) Find the acceleration  $\mathbf{a}(t)$ .

(b) Find the unit tangent  $\mathbf{T}(t)$ , the unit normal  $\mathbf{N}(t)$  and the curvature  $\kappa(t)$ .

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(c) What is  $s(t)$ , the distance travelled along the curve as a function of  $t$ ? Assume that at  $t = 0$ ,  $s = 2$ .

(d) Using your answers from part (b) and (c) verify the relation:

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}.$$

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3) Consider the function

$$f(x, y) = \frac{x}{x^2 - y^2}.$$

(a) What is

$$\lim_{(x,y) \rightarrow (1,1)} f ?$$

(b) Find  $\nabla f$ . What is the direction of steepest **decrease** of  $f$  at  $(1, 0)$ ?

(c) Find  $D_j f$  at the point  $(0, 1)$ . From the result, what can you say about the level curve of  $f$  at  $(0, 1)$ ?

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4) Find the following limits, or show that they do not exist.

(a)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy(x-y)}{(x^2+y^2)^{3/2}}$$

(b)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2+y^2}{\sin(2x^2+2y^2)}$$

(c) Is the function  $f = |xy|$  differentiable at  $(0, 0)$ ? Explain.

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5) Consider the quadric surface

$$x^2 + 4y^2 - z^2 = 1$$

(a) What is the equation for the tangent plane to the surface at  $(1, 1/2, 1)$ ?

(b) What is the parametric equation of the tangent line to the surface (at the same point) which is parallel to the  $yz$  plane?