MATHEMATICS 234–CALCULUS III–FINAL

Wednesday, December 20, 2006

Instructor: Paul Milewski

NAME (print): ____

Instructions:

Circle the name of your TA from the list below.

GUO PANTEA RAULT SOLOMOU

Show all your work. Partial credit is given only if your work is clear. Time allowed: 2 hours. GOOD LUCK!

1. ________(25)

2. ________(25)

3. ________(25)

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5. ________(25)

6. ________(25)

7. ________(25)

8. ________(25)

Total. ________(200)

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1) What is the minimum distance from the cone $z^2 = x^2 + y^2$ to $(1, 2, 0)$? What is the point on the cone closest to $(1, 2, 0)$? Justify why the result you obtained is indeed a minimum. [25]

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2) Consider the path of a particle parametrically written as

$$\mathbf{r}(t) = \frac{2}{3}t^{3/2}\,\widehat{\mathbf{i}} + \cos t\,\widehat{\mathbf{j}} + \sin t\,\widehat{\mathbf{k}}.$$

(a) What is s, the distance travelled along the curve as a function of t? Assume that at $t = 0$, $s = 0$. [5]

(b) What is the velocity $\mathbf{v}$ of the particle and the unit tangent to the curve $\widehat{\mathbf{T}}$? [10]

(c) What is the unit normal $\widehat{\mathbf{N}}$ and the curvature κ of the curve? [10]

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3) Consider the vector field in the plane

$$\mathbf{U} = \frac{\mathbf{r}}{|\mathbf{r}|^2} = \frac{x\widehat{\mathbf{i}} + y\widehat{\mathbf{j}}}{x^2 + y^2}$$

Using whatever method is applicable, find

(a) The flux of $\mathbf{U}$ through the circle of radius 1 centered at $(2, 0)$? [10]

(b) The flux of $\mathbf{U}$ through the circle of radius 1 centered at the origin ? [10]

(c) What is a general rule for the flux of this particular vector field $\mathbf{U}$ through an arbitrary closed curve in the plane? [5]

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4) Consider the vector field in the plane

$$\mathbf{F} = \mathbf{r}|\mathbf{r}|^2 = (x^3 + xy^2)\widehat{\mathbf{i}} + (yx^2 + y^3)\widehat{\mathbf{j}}$$

(a) Is the vector field conservative? If so calculate its potential function. [10]

(b) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the curve C comprised of the 2 straight line segments. The first segment is from $(0,0)$ to $(1,0)$ and the second from $(1,0)$ to $(3,2)$. [10]

(c) Calculate $\oint_T \mathbf{F} \cdot d\mathbf{r}$ for the closed triangle T with vertices $(0,0)$, $(1,0)$, $(3,2)$ taken *clockwise*. [5]

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5) Calculate

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{N} \, dS,$$

where $\mathbf{F} = yz\,\widehat{\mathbf{i}} + (z-x)\,\widehat{\mathbf{j}} - (y-x)\,\widehat{\mathbf{k}}$ and S is the part of the ellipsoid $9x^2 + y^2 + z^2 = 1$ where $x \geq 0$ and $\mathbf{N}$ is the *outward* normal of the ellipsoid. [25]

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6) (a) Find the equation for the tangent plane to the sphere $x^2 + y^2 + z^2 = 9$ at the point $(1, 2, 2)$. [10]

(b) Find the volume of the solid in the first octant between the sphere and the plane found in (a). [15]

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7) Let

$$\begin{aligned} z &= f(x,y) = x^2 + y^2, \\ x &= g(s) = \frac{3}{5}\cos(s), \\ y &= h(s) = \frac{4}{5}\sin(s). \end{aligned}$$

(a) Find $\frac{dz}{ds}|_{s=\pi}$. [8]

(b) Find $D_{\hat{\mathbf{u}}}f(-\frac{3}{5}, 0)$ where $\hat{\mathbf{u}} = -\hat{\mathbf{j}}$. [8]

(c) Sketch the curve $(x(s), y(s))$. With your sketch, describe in less than 20 words what the results of (a) and (b) represent. [9]

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8) Find the volume contained in the intersection of the solid spheres $\rho = 2$ and $\rho = 2\sqrt{2}\cos\phi$. Sketch the spheres. [25]

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SCRATCH WORK

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SCRATCH WORK