Math 234 Final
Date: May 18th, 2007
Instructor: Yong-Geun Oh

No calculator allowed.
Write detailed work to obtain full credits.

Your Name:

Please circle your TA's name :

Weiyong He Seth Meyer Jie Ling

Problem/pt	Score
P1/40	
P2/30	
P3/20	
P4/20	
P5/40	
P6/30	
P7/20	
Total/200	

1. **(10 points each) No partial credits for these problems!**

   (a) Change the Cartesian coordinates $(x, y, z) = (\frac{1}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}, -\frac{\sqrt{3}}{2})$ to spherical coordinates (ρ, θ, ϕ) with $\rho \geq 0$, $0 \leq \theta < 2\pi$, $0 \leq \phi \leq \pi$.

   (b) Evaluate the integrals

   $$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 3x \, dydx.$$

   (c) Sketch the region of integration in the xy-plane.

   (d) Evaluate the iterated integral

   $$\int_{-1}^{2} \int_0^{3x} \int_{x+2}^{y} 4 \, dz \, dy \, dx$$

2. Evaluate the following integrals

   (a) **(10 pts)** $\int_C (2x+y)\,dx+(x-y)\,dy$; C is the line segment from $(1,1)$ to $(3,-1)$.

   (b) **(10 pts)**

$$\int_C \frac{x\,dy - y\,dx}{x^2+y^2};$$

   C is the unit circle with its center at the origin with counterclockwise orientation.

   (c) **(10 pts)** By changing the order of integration, evaluate the integral

$$\int_0^4 \int_{x/2}^2 e^{y^2}\,dy\,dx.$$

3. **(20 pts)** Find the volume of the solid bounded by the cylinders $x^2 = y$, $(z-1)^2 = 4y$ and the plane $y = 3$.

4. Consider the function $f(x, y) = 2x^2y - xy^3$ and its graph $z = f(x, y)$.

(a) **(10 pts)** Find the linear approximation of the function at the point $(x, y) = (1, 1)$. **(Simplify your answer. No simplification will result in some points off!)**

(b) **(10 pts)** Estimate the maximal possible error for this linear approximation when the point (x, y) is chosen so that

$$|x - 1|, \, |y - 1| \leq \frac{1}{2}.$$

5. The following problem concerns Green's formula

(a) **(10 pts)** State precisely Green's formula for a vector field $\mathbf{F} = M\mathbf{i} + N\mathbf{j}$ on the plane in circulation-curl form. **(You should state clearly about the conditions on domain of integration and on the orientation of its boundary. This will also help you correctly solve the part (c) of this problem!)**

(b) **(10 pts)** Choose a suitable vector field $\mathbf{F}$ and prove using Green's formula that if R is a region in the place bounded by a piecewise smooth closed curve C with the boundary orientation in terms of the region R, then

$$\text{Area of } R = \frac{1}{2}\oint_C (x\,dy - y\,dx)$$

(c) **(20 pts)** Using the formula found in (b), find the area of the region bounded by the triangle with vertices $(1,1)$, $(-1,-1)$ and $(2,-2)$.

6. Consider the surface G in 3-space parameterized by

$$\mathbf{x}(s,t) = (s^2 + t^2, 2s, 2t)$$

on the domain given by the disc

$$s^2 + t^2 \leq 3$$

(a) **(10 pts)** Express the surface element dS in terms of ds, dt.

(b) **(20 pts)** Suppose the mass density of the surface is given by the function $\delta(x, y, z) = y^2 + z^2$. Find the mass of the surface. **(Be prepared for doing integration by substitution you learned in 222. Good substitution will simplify your calculation.)**

7. Consider the integral

$$\int_{(\frac{\pi}{2},-1,2)}^{(\frac{\pi}{3},1,0)} (yz + y\cos x)\,dx + (xz + \sin x)\,dy + (xy + 2)\,dz.$$

(a) **(10 pts)** Prove that the integrals do not depend on the choice of paths connecting the given end points.

(b) **(10 pts)** Evaluate the integrals.