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Mathematics 234

Midterm exam II

October 25, 2004

YOUR NAME:

Circle your TA's name: Brian Weber Mark Rummel Paul Johnson

There are four problems. You need to show all your work.

Problem 1 (max 25 pts.):

Problem 2 (max 25 pts.):

Problem 3 (max 25 pts.):

Problem 4 (max 25 pts.):

Total:

1. The iterated integral

$$\int_0^2 \left\{ \int_{x^2}^{2x} \frac{y}{16} dy \right\} dx$$

can be interpreted as an integral $\iint_{\Omega} y/16 \, dA$.

- (i) (20 pts.) Sketch the planar region Ω and reverse the order of integration.
- (ii) (5pts.) Evaluate the integral.

2. Let Ω be the planar region in the first quadrant ($x \geq 0, y \geq 0$) which is bounded by the circles of radii 1 and 4 centered at the origin and by the rays $y = 0$ and $y = x/2$. Sketch the region.

You are given the integral

$$\iint_{\Omega} x dA.$$

Set this integral up in polar coordinates and then compute it.

3. Consider the planar region which is given as

$$D = \{(x, y) : x = r \cos \theta, y = r \sin \theta, 0 \leq r \leq \sin \theta, 0 \leq \theta \leq \pi/4\}.$$

(i) Sketch D and compute its area.

(ii) Let S be the piece of the plane $3x + 2y + z = 1$ which lies above D , that is

$$S = \{(x, y, z) : (x, y) \in D, 3x + 2y + z = 1\}.$$

Compute the area of S .

Note: The trigonometric identity $2 \sin^2 \theta = 1 - \cos(2\theta)$ might be helpful to quickly evaluate an integral.

4. Consider the region Ω which is bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 1$. You are given the integral $\iiint_{\Omega} f(x, y, z) dV$. Set this up as an iterated integral in two ways,

(i) as $\int_{z=*}^* \int_{y=*}^* \int_{x=*}^* f(x, y, z) dx dy dz$,

(ii) and as $\int_{y=*}^* \int_{z=*}^* \int_{x=*}^* f(x, y, z) dx dz dy$.

You are not asked to compute the integrals.

Scratch paper.