

**Mathematics 234**

**Midterm exam II**

October 25, 2004

**NAME:**

**Discussion section or TA:**

There are four problems. You need to show all your work.

Problem 1 (max 30 pts.):

Problem 2 (max 30 pts.):

Problem 3 (max 30 pts.):

Problem 4 (max 10 pts):

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Total:

1. Assume the differentiable function of three variables,  $f(x, y, z)$ , satisfies

$$f(1, -2, 2) = 6, \quad f_x(1, -2, 2) = 2, \quad f_y(1, -2, 2) = 1, \quad f_z(1, -2, 2) = -3.$$

Using this information, answer (a)-(c):

(a) (10 pts.) Compute the value of the directional derivative of  $f$  at  $P = (1, -2, 2)$  in the direction toward the origin.

(b) (10 pts.) Suppose you are told that there is a direction  $u$  in which the directional derivative of  $f$  equals 4. Can this be true? If so, find such a direction  $u$ . If not, explain why not.

(c) (10 pts.) A particle is moving along a curve  $C$  parametrized by

$$\mathbf{r}(t) = (t - 1)\mathbf{i} + (t^2 - 6)\mathbf{j} + (t^3 - 6)\mathbf{k}$$

sees changing values of  $f(x, y, z)$ . Compute the rate of change  $\frac{d}{dt}(f(\mathbf{r}(t)))$ , at time  $t = 2$ .

2. Consider the surface defined by

$$(*) \quad x^2 + y^2 - z^2 = 2x(y + z) - 2.$$

(a) (15 pts). Find the equation of the plane tangent to the surface at the point  $P = (1, 2, 1)$  on  $S$ .

(b) (15 pts). The equation (\*) for  $S$  defines  $z$  as a function of  $x$  and  $y$ . Compute the partial derivative  $\frac{\partial z}{\partial y}(1, 2)$ .

3. The temperature at the point  $(x, y, z)$  of the sphere  $S : x^2 + y^2 + z^2 = 1$  is given by  $T(x, y, z) = xy + z^2 - z$ .

(a) (20 pts.) Find the hottest and coldest points on  $S$ . Be sure to consider all possibilities and to show your work.

(b) (10 pts.) Are there critical points for  $T$  inside  $S$ , i.e. for  $x^2 + y^2 + z^2 < 1$ ? If so, compare the temperature  $T$  at such points with the temperature on  $S$ . If we look inside and on the sphere  $S$ , where do the absolute extreme temperatures exist?

4. (10 pts.)

Consider the function given by

$$f(x, y) = (x - 1)^4 + (y - 2)^3.$$

Find every critical point. Does the function have a local minimum or a local maximum somewhere? Justify your answer.

Extra sheet for computations.