

## Math 320 Sample Second In-class Exam

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1. Decide if the sets  $S_1$  and  $S_2$  below are vector spaces or not. Show a basis for whichever of them is a vector space.

(a) (15 points)  $S_1$  is the set of  $\mathbf{A}$   $n \times n$  matrices for which  $\mathbf{A}^T = \mathbf{A}^{-1}$ .

(b) (15 points)  $S_2$  is the set of  $\mathbf{B}$   $2 \times 2$  matrices for which  $\mathbf{B} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ .

2. (40 points) By computing the Wronskian, decide if  $\sin x$ ,  $\cos x$ ,  $\tan x$  are linearly independent functions on the real interval  $(0, \frac{\pi}{2})$ .

3. (50 points) Solve the initial value problem

$$x''(t) - 4x'(t) + 3x(t) = \cos t, \quad x(0) = 0, \quad x'(0) = 0.$$

(Note that this is *not* an equation of a damped vibration as the speed term has a negative coefficient.)

4. Let

$$\mathbf{A} := \begin{pmatrix} 3 & \sqrt{3} & 2\sqrt{3} \\ \sqrt{3} & 1 & 2 \\ 2\sqrt{3} & 2 & 4 \end{pmatrix}.$$

(a) (20 points) Determine the eigenvalues of this matrix.

(b) (20 points) Show bases for each eigenspace corresponding to the eigenvalues.