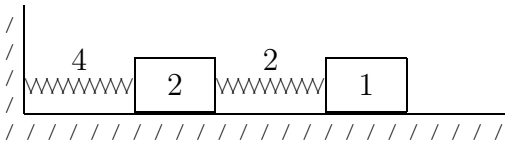
Math 320 Final Exam

May 11, 2004, 7:45 am, B239 Van Vleck
Márton Balázs **NAME:**

1. (60 points) Solve

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} = -3x_1 + 2x_2 \qquad x_1(0) = 1$$
$$\frac{\mathrm{d}x_2}{\mathrm{d}t} = -x_1 - x_2, \qquad x_2(0) = 1.$$

2. (50 points) Determine the natural modes and the corresponding frequencies of the following mass-spring system:



(Numbers above the springs represent the spring constants, and numbers in the bodies stand for the masses.)

3. (60 points) Use fundamental matrices and variation of parameters to solve

$$\begin{aligned}
\frac{\mathrm{d}x_1}{\mathrm{d}t} &= x_1 + 2x_2 + \mathrm{e}^{-t} && x_1(0) = 2 \\
\frac{\mathrm{d}x_2}{\mathrm{d}t} &= \quad 3x_2 + \mathrm{e}^{t}, && x_2(0) = 1.
\end{aligned}$$

4. (30 points) Use Gauss-Jordan elimination to solve the system

$$\begin{aligned} 3x_1 + x_2 &= 5 \\ x_1 + x_2 + 2x_3 &= 9 \\ 4x_1 - 2x_2 - x_3 &= -3. \end{aligned}$$

5. (40 points) Solve the initial value problem of the damped and forced resonance

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = \cos t, \qquad x(0) = 0,\ x'(0) = 0.$$

Also find the amplitude of the asymptotic motion (valid after a long time).