

Math 431 Second Evening Exam
Version B

Room B123, 5:00pm - 6:00pm, November 19, 2004
Márton Balázs

NAME:

1. In the post office there are two clerks, Mr. A and Mr. B. When I arrive, both of them have just started serving a customer, but I am next in line. Mr. A and Mr. B serve customers in X and in Y minutes, respectively, where X and Y are two independent exponential random variables, X with parameter 2, and Y with parameter 3.

(a) (15 points) What is the probability that I will have to wait t minutes before being served, i.e. the probability of $\{X > t, Y > t\}$? Show your computation, or explain your answer.

(b) (15 points) Using your answer in (a), what is the distribution of my waiting time before being served? What is my expected waiting time?

(c) (20 points) Compute the probability that I will be served by Mr. A, i.e. the probability of the event $\{X < Y\}$. Show your computation, or explain your answer.

(d) (10 points) Is it important that Mr. A and Mr. B *have just started* serving the customers? What if they had been serving them for the last two and six minutes, respectively?

2. Assume a reservoir of 1 000 gallons gets filled every time a rainfall happens, and has $1\,000 e^{-t/3}$ gallons of water t time after the last rainfall (where t is measured in weeks). Times passing between consecutive rainfalls are independent, and the time T between two rainfalls is an exponentially distributed random variable with parameter $\frac{1}{3}$ (weeks⁻¹).

(a) (30 points) Let $X = 1\,000 e^{-T/3}$ be the amount of water in the reservoir just when the next rainfall begins. Then X is a random variable. Compute its distribution function and density function. What kind of random variable is it? What is its expectation?

(b) (30 points) What is the probability that there are no rainfalls on my nine-week vacation? What is the expected number of rainfalls during that time? What is the probability that there are at least three rainfalls during that time?

(c) **Bonus problem** (only try when all other problems are completed and checked, additional 5 points): Compute *on average* what percent of the amount of water, measured on June 1 at 8:00am, is left by the time of the first rainfall after June 1, 8:00am. Compute also what percent of the amount of water, measured on June 1 at 8:00am, is left by the *expected time* of the first rainfall after June 1, 8:00am. Which is larger?

3. (40 points) Squares produced in the square factory have edges of length X . We know that the average length of the edges is 3 inches, and the average area of the squares is 11.25 inches². If X is approximately normally distributed, then what is the probability that a square has edges shorter than 2.25 inches? (The normal distribution is to be found on the last page.)