Math 431 Final Exam

Room B239, 2:45pm - 4:45pm, December 18, 2004
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1. An insurance company runs a review on their payments, and reaches the following conclusions:

- 5% of policyholders had at least one accident in 2001, 5% of policyholders had at least one accident in 2002, and 5% of policyholders had at least one accident in 2003.
- 1% of policyholders had at least one accident in both 2001 and 2002, 1% of policyholders had at least one accident in both 2001 and 2003, and 1% of policyholders had at least one accident in both 2002 and 2003.
- 0.5% of policyholders had at least one accident in each 2001, 2002, and 2003.

(a) (15 points) Compute the probability that a policyholder has at least one accident in 2002, given he/she had at least one in 2001. Is the probability of this policyholder having an accident in 2002 higher then the probability of the other policyholders having one in 2002? Are accidents of policyholders occurring independently in these two years?

(b) (25 points) What percentage of policyholders had no accident at all during 2001 – 2002 – 2003? *Hint: don't forget your yes or no answer for (a)!*

(c) **Bonus problem** (only try if all other problems are completed and checked, 10 points): Compute all kind of conditional probabilities in time: What is the probability that a policyholder has at least one accident in 2002, given he/she

- had at least one in 2001 (same as question (a) above),
- did not have any in 2001?

What is the probability that a policyholder has at least one accident in 2003, given he/she

- had at least one in each 2001 and 2002,
- had at least one accident in one of the years 2001 and 2002, but did not have any in the other year,
- did not have any accidents in 2001 and 2002?

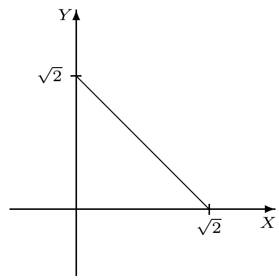
2. (40 points) Let X be a uniform random variable on $(0, 1)$. Determine first the possible values, and then the probability density function of

$$Y := \frac{1}{X} - 1.$$

What can be said about $\mathbb{E}(Y)$?

3. (40 points) Two fair dice are rolled. Find the probability mass function of the difference between the numbers shown on the two dice. Also find the expected value of the difference.

4. Let X and Y be the two coordinates of a point uniformly distributed over the following triangle:



(a) (10 points) What are the marginal distribution functions of X and of Y? (Hint: draw!)

(b) (10 points) What is the conditional distribution of Y given $X = x$?

(c) (5 points) Are X and Y independent? Are they identically distributed?

(d) (10 points) Compute $\mathbb{E}(Y \mid X = x)$ and $\mathbf{Var}(Y \mid X = x)$.

(e) (10 points) Compute $\mathbb{E}(X)$ and $\mathbf{Var}(X)$, $\mathbb{E}(Y)$ and $\mathbf{Var}(Y)$.

(f) (10 points) Compute the correlation $\mathbf{Corr}(X, Y)$.

(g) (15 points) Using the above expectation, apply Markov's inequality to bound $\mathbb{P}(Y > 1)$.

(h) (10 points) Compute the exact value of $\mathbb{P}(Y > 1)$, and compare it to your answer for (g).

5. (40 points) We are given 100 coins, each of them can independently be a penny, a nickel, a dime, or a quarter with equal chance. Estimate the probability that the joint value of our coins exceeds $\$\,10$. Give a numerical answer.