Math 431 Sample First Evening Exam

Room B239, 6:15 - 7:15pm, February 23, 2004
Márton Balázs

1. (45 points) "*Aoccdrnig to a rscheearch at an Elingsh uinervtisy, it deosn't mttaer in waht oredr the ltteers in a wrod are, the olny iprmoetnt tihng is taht frist and lsat ltteer is at the rghit pclae. The rset can be a toatl mses and you can sitll raed it wouthit porbelm. Tihs is bcuseae we do not raed ervey lteter by itslef but the wrod as a wlohe.*"

Based on this research, let's assume that an English word is uniquely determined by its first and last letter, and the unordered set of its other letters. Put it in another way, given an English word, changing the order of the inner letters (and so leaving the first and last letter fixed) cannot lead to a different English word. Give then an upper bound on the number of five-letter English words, and compare it to the number of all possible sequences of length five, composed of any of the 26 letters of the alphabet.

2. (35 points) Days in the year can be either cold or hot and, regardless of this, either sunny or cloudy. If half of the days in the year are hot, 2/3 of them are sunny, and 2/5 of them are both hot and sunny, then what is the number of both cold and cloudy days?

3. (35 points) Once upon a time Odysseus met an intersection of three pathways. One of them lead to Athens, the other lead to Mycenae, and the third lead to Sparta, but he didn't know which route goes to which of these cities. He chose one of the routes by rolling a die, giving equal chance to each of these choices. He knew that, on average, Athenians only tell the truth in one case out of three, Mycenae citizens lie every second time, but people of Sparta are always honest. In the city he arrived, he asked the first man he met how many two times two was, and had four as answer. What is the probability that Odysseus finally reached Athens?

4. We choose a random integer number from the set $\{1, 2, \ldots, n\}$, where $n > 1$ is a fixed positive integer. Let E denote the event that the number we chose is even, and let F denote the event that this number is divisible by 3.

(a) (20 points) Show that E and F are independent events in case n is divisible by 6.

(b) (25 points) Show that these events are also independent if the remainder of n when divided by 6 is two.