Math 431 Sample Second Evening Exam

Room B239, 6:15 - 7:15pm, April 13, 2004
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1. When I'm riding my motorcycle, I have, on average, three bugs splashed on my helmet every mile.

(a) (10 points) What is the probability that there will be no bug on my helmet for the first mile?

(b) (15 points) What is the probability that there will be no bug on my helmet for the first half of a mile?

(c) (15 points) After how many miles will I have at least one bug on my helmet with probability 1/2?

2. (40 points) Based on earlier experiences, I know that each friend I invite to my party comes independently with probability 50%. How many of my friends can I invite if I want to have at most 30 people with probability at least 90%? Use normal approximation to answer this question.

3. (35 points) Let X be a uniform random variable over the real interval (0, 1). Define

$$Y := \frac{-\ln(1-X)}{\lambda}.$$

Compute the probability density function of Y. What type of distribution does Y have? (Remark: this is the way a computer can generate this type of random variable. This is also the way one can generate this distribution using a simple calculator equipped with an RND button.)

4. (45 points) We break a stick at two random, uniformly and independently chosen points. What is the probability that considering the three parts obtained as edges, a triangle can be assembled?

Please turn over for the standard normal distribution.