

Math 431 Sample Final Exam

Room B130, May 10, 2004, 2:45pm
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1. (50 points) A street has n street lamps, and it can be divided into $n - 1$ intervals between the neighboring lamps. Such an interval is said to be dark, if both lamps at its ends are defective. Use indicator variables to compute the expected number of dark intervals, if each lamp is defective independently with probability p .

Bonus question: (10 points) Compute the variance of the number of dark intervals on that street.

2. Let X be a uniformly distributed random variable on the real interval $(0, 1)$. Given X , let the conditional distribution of Y be uniform on the interval $(0, X)$. Compute

a) (30 points) the correlation of X and Y ,

b) (30 points) the conditional expectation $\mathbb{E}(X|Y = y)$.

3. (40 points) Use the Central Limit Theorem to estimate the probability that one has to roll a die more than 300 times for the sum of the rolls to reach 1000.

4. (30 points) On a long trip, the probability of having a blow-out on the first wheel of my bicycle is 0.1. The probability of having a blow-out on the rear wheel is also 0.1. However, these events are not independent. Show that the probability that I will have no blow-out during the trip is at least 0.8.

5. (30 points) Let X be an exponential random variable, and define $Y = e^X$. Compute the probability density function of Y .

6. (30 points) In a school, classes consisting of 15 boys and 10 girls are started. The number of blue-eyed girls in such a class is a binomial random variable with parameters $(n = 10, p = 0.25)$. The number of blue-eyed boys is also a binomial random variable with parameters $(m = 15, p = 0.25)$. Compute the probability that exactly 8 blue-eyed children will go to a given class.