Math 475 Sample First In-class Exam

October 13, 2003
Márton Balázs

1. (5 points) We only know about Pigeon Airlines that they have three types of airplanes: small ones, medium-sized ones and large ones, and that the number of all their airplanes is 32. For a special occasion we need planes of the same type: either 15 small planes, or 12 medium-sized planes, or 8 large planes. Are they for sure able to provide the airplanes we need?

2. (5 points) Today we have 8 things to do in the office, namely, jobs numbered $1, 2, \ldots, 8$. However, job number 4 can only be done after job number 2 is completed. In how many orders is it possible to complete our jobs?

3. In the three dimensional space, a walker starts from the origin (0, 0, 0). At each step, she can step either one to the right (adding one to her x-coordinate), or one behind (adding one to her y-coordinate), or up (adding one to her z-coordinate).

a) (7 points) At how many possible positions can she finish her walk after 10 steps?

b) (5 points) Assume she ended her walk at (3, 5, 2). On how many different paths could she get there?

4. Answer the following questions:

a) (5 points) Which is the permutation with inversion sequence 4, 2, 1, 0, 0?

b) (5 points) What is the sign of the permutation $(3, 5, 4, 1, 2)$?

5. Define the relation R on the non-empty subsets of $\{1, 2, 3, 4\}$ by XRY if and only if either the smallest element of subset X is smaller than the smallest element of subset Y, or $X = Y$.

a) (5 points) Show that R is a partial order on the non-empty subsets of $\{1, 2, 3, 4\}$.

b) (3 points) Represent R by its diagram.

c) (2 points) Show a linear extension of R.

6. Answer the following questions:

a) (3 points) The coefficient of x^4 in the extension of $(1+x)^n$ is 15. What is the value of n?

b) (5 points) Let p, q be positive numbers such that $p+q=1$. Compute

$$\sum_{k=0}^{n} \binom{n}{k} \cdot k \cdot (k-1) \cdot p^k \cdot q^{n-k} \quad .$$

7. (10 points) How many of the 2^6 combinations of the set $\{1, 2, 3, 4, 5, 6\}$ has no three neighboring elements?
(Hint: for $1 \leq i \leq 4$, let A_i be the set of combinations containing i, $i+1$, $i+2$ and use the Inclusion-Exclusion Principle.)