

## Math 475 Second In-class Exam

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**NAME:**

1. (12 points) We have a quantity  $h$ , starting from initial value  $h_0$ . We want to approach the value 100 with this quantity. Therefore, we perform steps to adjust the value  $h$  from  $h_0$  towards the desired value 100. We fix a constant  $A$  and at each step we obtain our new value  $h_{n+1}$  by increasing the previous value  $h_n$  by  $A$  times its difference to 100. Show the exact set of  $A$ 's for which this method makes  $h_n$  converge to 100, and describe what happens for  $A$ 's not in this set.

2. (12 points) Solve the recursion

$$h_n = h_{n-1} + 2h_{n-2} + n + \frac{3}{2}$$

with initial values  $h_0 = 2$ ,  $h_1 = \frac{5}{2}$ . Bonus question (additional 6 points): find the generating function for the solution sequence  $h_n$ .

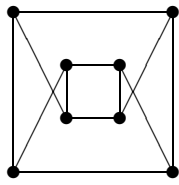
**3.** (12 points) 100 women and 99 men are invited to a party. After having waited in the foyer, they enter the saloon one by one. Use the Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

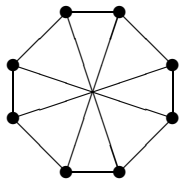
to compute the number of orders they can enter the saloon such that at each moment there is strictly more women than men in the saloon. (Don't forget at the end that people are distinguishable.)

4. (12 points) Assume that the non-general graph  $G$  has 10 vertices, one of them with degree 6, and 11 edges. Show that there is no Hamilton cycle in  $G$ . Is there such a graph with an Eulerian cycle? And with an Eulerian trail?

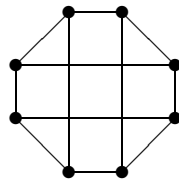
5. (12 points) Consider the following four connected cubic graphs of order 8:



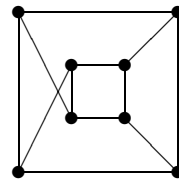
$G_1$



$G_2$



$G_3$



$G_4$

For each pair of them, either show an isomorphism or prove that they are not isomorphic. Show that two of  $G_1, G_2, G_3, G_4$  are planar graphs.