

## Math 475 Final Exam

December 14, 2003, 7:45 am  
Márton Balázs

NAME:

1. (10 points) Compute the number of different coin-purses containing altogether 20 coins of pennies, nickels, dimes and quarters.

2. a) (3 points) Find the inversion sequence of the permutation (1 6 4 5 3 2).

b) (3 points) Find the permutation which has the inversion sequence 4, 4, 2, 2, 1, 0.

c) (3 points) Find the sign of the permutations in a) and b).

3. (7 points) Compute the sum

$$\sum_{k=0}^n \binom{n}{k} (x-1)^k .$$

4. (12 points) Solve the linear recurrence

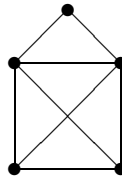
$$h_n = 2h_{n-1} + 3h_{n-2} + 4 \times (-1)^n$$

with initial values  $h_0 = 0$ ,  $h_1 = -1$ .

**Extra bonus problem:** (additional 6 points, try only when you have the solution for 4.) Find the generating function *and* the exponential generating function for the solution  $h_n$  obtained for problem 4. above.

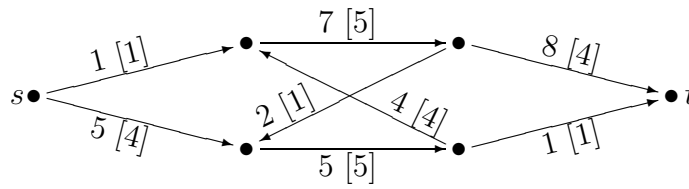
5. (12 points) Use exponential generating functions to find the number of  $n$  digit base three numbers (i.e. numbers made of digits 0, 1, 2) containing the digit 1 an even number of times. (Remark: these are exactly the  $n$  digit base three numbers, which are even.)

6. (10 points) Compute the chromatic number and the chromatic polynomial of the following graph of order 5:



7. (10 points) Let  $G$  be a (non-general, non-directed) connected graph such that each vertex has an even degree. Show that  $G$  has a strongly connected orientation. (Hint: think about a special thing the graph  $G$  does have.)

8. In the following network, capacities are represented by numbers, and values of a function  $f$  are represented by numbers in brackets  $[\cdot]$ .



a) (3 points) Show that  $f$  is a flow from  $s$  to  $t$ , and determine its value.

b) (4 points) Use the Basic Flow Algorithm (as many times as needed) to generate a maximum flow  $f'$  from  $f$ .

c) (3 points) Give a minimum cut and compute its capacity.

9. (10 points) Use Burnside's theorem to compute the number of inequivalent colorings of the 6 vertices of the following rigid frame (subject to turning in any direction in the three dimensional space), using three colors:

