Math 475 Sample Final Exam

December 14, 2003, 7:45 am
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1. (10 points) Consider the following relations R on the different sets X, and decide for each if it is an equivalence relation, partial order, total order, or none of these.

a) G is a graph with two pendent vertices u and v, and each other vertex has degree 2, X is the set of vertices. For vertices x, y, xRy iff for their distances from u we have $d(x, u) \leq d(y, u)$.

b) G is a graph with a vertex u, X is the set of vertices, and for vertices x, y, xRy iff $d(x, u) < d(y, u)$ or $x = y$.

c) $X = \mathbb{R}$ the set of real numbers, and xRy iff $x^4 = y^4$.

d) G is a graph, X is the set of vertices, and for x, y vertices xRy iff there is a walk from x to y.

e) D is a digraph, X is the set of vertices, and for x, y vertices xRy iff there is a path from x to y.

2. (6 points) Compute the value of $\sum_{k=1}^{n} k(-2)^k \binom{n}{k}$.

3. (10 points) Determine the number of bags containing altogether n fruits of apples, bananas and pears, such that

- the number of apples is even, and is not more than 6,
- the number of bananas is a multiple of 8, and
- the number of pears is 0 or 1.

4. (10 points) Explain how the recurrence relation

$$s(p,\, k) = (p-1)s(p-1,\, k) + s(p-1,\, k-1) \qquad (1 \leq k \leq p-1)$$

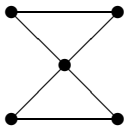
and the initial values $s(p,\, 0) = 0 \quad (p \geq 1)$, $s(p,\, p) = 1 \quad (p \geq 0)$ for the Stirling numbers of the first kind can be obtained by combinatorial reasoning and by knowing what they count.

5. (10 points) Show that

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

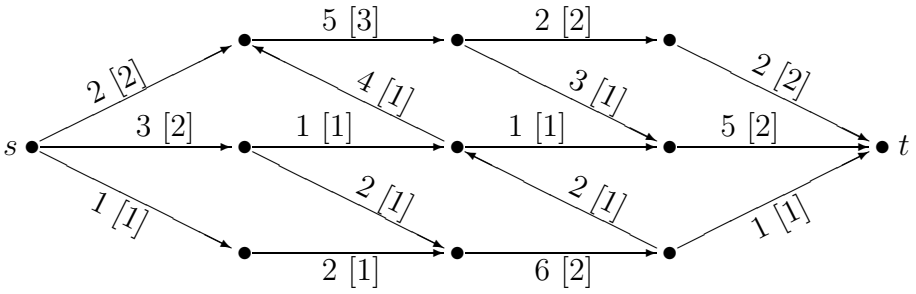
is an adjacency matrix of a non general graph. Is this graph a planar graph? If yes, then show a planar representation for it.

6. (12 points) Compute the chromatic polynomial and the chromatic number for the following graph:



7. (8 points) Show that a strongly connected tournament is not transitive.

8. (12 points) In the following network, capacities are represented by numbers, and values of a function f are represented by numbers in brackets $[\cdot]$.



a) Show that f is a flow from s to t, and determine its value.

b) Use the Basic Flow Algorithm (as many times as needed) to generate a maximum flow f' from f.

c) Give a minimum cut and compute its capacity.

9. (12 points) Use Burnside's theorem to compute the number of inequivalent colorings of the 5 vertices of the following rigid frame (subject to turning in any direction in the three dimensional space), using three colors: