

Answers to 2003 Fall Semester Sample Final Exam

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1.

- (a) Total order.
- (b) Partial order.
- (c) Equivalence.
- (d) Equivalence.
- (e) None, in general.

2.

$$2n \cdot (-1)^n.$$

3.

Precisely one way for each $n \geq 0$.

4.

See Theorem 8.2.9 in your notes or in the book.

5.

Vertices 2 to 5 and edges between them form a K_4 , vertex 1 is connected to vertices 2 and 5. Planar representation comes from the one of K_4 , see Figure 11.4 in the book.

6.

The chromatic number $\chi(G) = 3$, the chromatic polynomial is $p_G(k) = k(k-1)^2(k-2)^2$.

7.

As it is strongly connected, there cannot be a consistent ranking.

8.

- (a) The value of f is 5.
- (b) There is one path from s to t where one can increase the value of f by one, and reach a value of 6.
- (c) A minimum cut consists of the three arcs leaving s . It's capacity is 6.

9.

There are 63 inequivalent colorings.

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1.

There are 1771 ways.

2.

(a) The inversion sequence is 0, 4, 3, 1, 1, 0.

(b) The permutation is (6, 5, 3, 4, 1, 2).

(c) Both have negative sign.

3.

The sum equals x^n .

4.

$h_n = n \cdot (-1)^n$. Bonus: $g(x) = \frac{-x}{(1+x)^2}$, $g^{(e)}(x) = -xe^{-x}$.

5.

The number of such numbers is $h_n = \frac{1}{2}(3^n + 1)$.

6.

The chromatic number $\chi(G) = 4$, the polynomial is $p_G(k) = \frac{k!}{(k-4)!}(k-2)$.

7.

G has then an Eulerian cycle, which can be oriented resulting in a strongly connected digraph.

8.

(a) The value of f is 5.

(b) There is one path from s to t where increasing the flows in the t direction and decreasing them in the s direction by one is possible. The result is a maximum flow with value 6.

(c) A minimum cut is one including the two edges leaving s . Its capacity is 6.

9.

There are 216 inequivalent colorings.