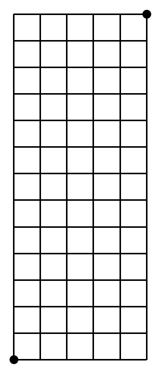
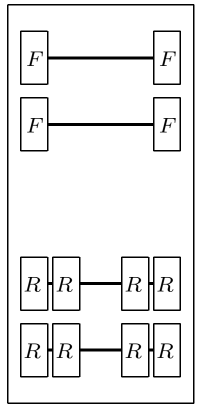
Math 475 first evening exam

3:30 - 4:30 pm, October 29, 2004
Márton Balázs **NAME:**

1. (10 points) The map of a city looks like a square grid. A bike race is organized on the streets of this city, and the finish is 5 blocks east and 13 blocks north from the start. The organizers decide to choose one of the shortest paths possible, so the total length to go will be 18 blocks. Show that there will necessarily be at least three consecutive blocks in the northerly direction somewhere along the path of this race.



2. A heavy truck has four front wheels (F) and eight rear wheels (R) in the following configuration:



All 12 tires are removed for general maintenance. In how many ways can they be put up again, if

(a) (5 points) any tire can go to any wheel,

(b) (5 points) front (F) and rear (R) tires are not to be mixed up,

(c) (5 points) front (F) and rear (R), and also left and right tires are not to be mixed up?

3. (10 points) 30 students are to be transported in a white car of 4 passenger seats, a black car of 4 passenger seats, an SUV of 6 passenger seats, and two **identical** vans of 8 passenger seats. In how many ways can the students be distributed to these vehicles?

4. (20 points) Let P be the set of permutations of $\{1, 2, 3\}$. For $p, q \in P$, let $a_1^{(p)}, a_2^{(p)}, a_3^{(p)}$, and $a_1^{(q)}, a_2^{(q)}, a_3^{(q)}$ be the inversion sequence of p and q, respectively.

- Let R be a relation on P defined by pRq if and only if the signs of p and q agree.
- Let R' be a relation on P defined by $pR'q$ if and only if $a_i^{(p)} \leq a_i^{(q)}$ for each $i = 1, 2, 3$.

Is R and R', respectively, an equivalence relation, a partial order, or neither? Why?

Show the equivalence classes of the equivalence relation(s) (if any).

Draw the diagram and show a linear extension of the partial order(s) (if any).

5. 5 identical lemon drinks and 5 identical orange drinks are to be distributed among 6 thirsty students.

(a) (10 points) In how many ways can this be done?

(b) (15 points) Define the set A_i, $i = 1, \ldots, 6$ to be the set of distributions with student i not receiving any drinks. Use inclusion-exclusion on these sets to compute the number of ways to distribute the drinks if each student receives at least one drink.