632 Introduction to Stochastic Processes Spring 2004
Midterm Exam II

Instructions: Show calculations and give concise justifications for full credit. Points add up 100.

1. (20 pts) Eileen is catching fish at the Poisson rate of λ per hour. Each fish is a salmon with probability 1/3 and a trout with probability 2/3. Eileen fishes for 1 hour. Find the probability that she catches exactly 2 trout and at least 1 salmon. (Try to give the simplest expression you can.)

2. (20 pts) Given that a rate λ Poisson process has exactly two arrivals in time interval $[0, t]$, compute the mean of the second (later) arrival time. In symbols, find the expectation $E[\, T_2 \,|N(t) = 2]$.

3. (15 pts) Consider an M/M/1 queue where customers arrive as a Poisson process with rate λ and services happen at rate μ. The system starts empty, and then customers start arriving one by one. Find the probability that the first customer is still in service when the second customer arrives in the system.

4. (15 pts) Consider an M/M/1 queue where customers arrive as a Poisson process with rate λ and services happen at rate μ. Suppose that customers are "impatient" in this sense: each customer in queue but not yet in service leaves the system with rate β independently of the other customers. Let again X_t be the number of customers in the system, a Markov chain with state space $\{0, 1, 2, 3, \ldots\}$. Give the rates of this Markov chain.

5. Let α, β and γ be positive constants. Consider a continuous-time Markov chain X_t on the state space $S = \{1, 2, 3\}$. The rates are

$$q(1,2) = \alpha,\ q(1,3) = \beta,\ q(2,1) = \gamma,\ q(2,3) = q(3,1) = 1, \text{ and } q(3,2) = 0.$$

Let the Markov chain X_t start in state 1.

(a) (15 pts) Find the probability that the first jump of the chain happens before time $t = 5$.

(b) (15 pts) Find the probability that, after the initial state 1, the next two states visited are 2 and 3 in that order.